$\mathcal{N} = 3$ field theories in four dimensions

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$\mathcal{N} = 3$ SCFTs

Many examples of $\mathcal{N} = 0, 1, 2, 4$ CFTs are known, both Lagrangian and non-Lagrangian. But no example of $\mathcal{N} = 3$ SCFT (which was not $\mathcal{N} = 4$) was known until our work.[1] In fact, they were widely thought not to exist!

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Theorem: Every non-gravitational CPT-invariant $\mathcal{N} = 3$ Lagrangian is automatically $\mathcal{N} = 4$.

Minimal $\mathcal{N} = 3$ multiplet: $\{A_\mu(+1), 3\lambda(+\frac{1}{2}), 3\phi(0), \lambda(-\frac{1}{2})\}$. Its CPT-conjugate changes the helicities, completing the content into a $\mathcal{N} = 4$ multiplet. One can also see that the only Lagrangian with $\mathcal{N} = 3$ automatically has $\mathcal{N} = 4$.

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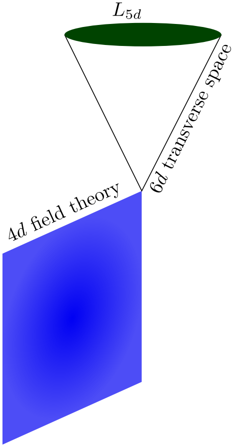
Theory engineering

We are after a theory in four dimensions which, if it exists, has no semi-classical limit compatible with the $\mathcal{N} = 3$ symmetry.

It turns out that the most robust way of constructing the 4d $\mathcal{N} = 3$ theories is by using string theory techniques in 10 and 11d.

More precisely, we will construct a string setting in 10d with a topological defect. On the core of this defect we will have a four dimensional theory coupled to 10d supergravity. In the IR the 10d supergravity decouples, leaving the 4d theory we are after.



IIB string theory

A good setting for our purposes is IIB string theory (described by type IIB supergravity at low energies). It contains certain supergravity defects ("D3-branes") where $\mathcal{N}=4$ four dimensional $U(N)$ SYM lives.

Furthermore, it has a scalar field τ_{10d}, whose restriction to the D3s gives the $\tau=\theta+i/g^2$ in the $\mathcal{N}=4$ Lagrangian.

Montonen-Olive duality on the low energy theory on the D3s

$$\mathcal{T}(U(N),\tau)=\mathcal{T}\left(U(N),-\frac{1}{\tau}\right) \tag{1}$$

extends to the full 10d string theory:

$$\text{IIB}(N\,\text{D3s},\tau_{10d})=\text{IIB}\left(N\,\text{D3s},\ -\frac{1}{\tau_{10d}}\right) \tag{2}$$

S-duality orbifolds (“S-folds”)

We want to somehow reduce $\mathcal{N} = 4 \to \mathcal{N} = 3$. It turns out that this is possible, by taking an appropriate quotient of the 10d theory.

Choose an element g of the Montonen-Olive duality group. It acts on the supercharges $Q_\alpha^{I=1,\ldots,4}$ as [Kapustin, Witten]

$$Q_\alpha^I \to \gamma(g) Q_\alpha^I \tag{3}$$

Simultaneously, act with a $U(1) \subset SO(6)$ rotation r on the $\mathbb{R}^6$ transverse to the D3s. This acts on the supercharges as

$$(Q_\alpha^1, Q_\alpha^2, Q_\alpha^3, Q_\alpha^4) \to (rQ_\alpha^1, rQ_\alpha^2, rQ_\alpha^3, r^{-3}Q_\alpha^4)\,. \tag{4}$$

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Crucially, generically this only makes sense for specific $\tau \in \{i, e^{2\pi i/3}\}$ for which $g(\tau)=\tau$. This means that the weak coupling limit is projected out!

F-theory viewpoint: Probing rigid singularities

From a string theory point of view, we will be interested in understanding the four dimensional physics coming from (probe D3 branes on) F-theory compactifications in the presence of singularities that do no admit supersymmetric smoothings. I.e. they cannot be resolved or deformed into a smooth space without spending energy.

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- Complex codimension 4 Calabi-Yau singularity in a geometry with a F-theory limit. There are many such geometries, and we will only scratch the surface.
- Simplest case: $\mathbb{Z}_k$ orbifolds of $\mathbb{C}^3 \times T^2$, with non-trivial T^2 action and isolated fixed points.

(Such orbifolds have appeared for two-folds [Dasgupta, Mukhi '96] and threefolds [Witten '96], but in these cases they are deformable.)

Generalizing the O3 plane

Calabi-Yau fourfolds of the form $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$ can be classified completely: the orbifold actions preserving susy were classified in [Morrison, Stevens '84], [Anno '03], [Font, López '04]. We focus on the cases preserving at least 12 supercharges.

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In the F-theory limit, adding D3 brane probes:

- $k = 1$ gives IIB string theory $\rightarrow$ 4d $U(N)$ $\mathcal{N} = 4$ SYM.
- $k = 2$ gives IIB w/ O3 plane $\rightarrow$ 4d $\mathcal{N} = 4$ SYM w/ orthogonal or symplectic groups.

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- $k = 3, 4, 6$ give IIB w/ exotic "OF3" plane $\rightarrow$ 4d $\mathcal{N} = 3$ SCFTs.

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- $k = 3, 4, 6$ give IIB w/ exotic "OF3" plane $\rightarrow$ 4d $\mathcal{N} = 3$ SCFTs.
  - [Ferrara, Porrati, Zaffaroni '98] propose a construction of exotic AdS_5 holographic backgrounds preserving $\mathcal{N} = 6$, similar to the expected form of the holographic dual of the $\mathcal{N} = 3$ SCFTs we find.
  - In [Aharony, Evtikhiev '15] some properties of these theories were understood, assuming they existed, but no construction was known.

Outline

EYAWTK about the O3 plane

It will prove very illuminating to revisit the O3 plane (i.e. $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$) from multiple viewpoints, since it is the simplest case of a complex codimension four singularity with a F-theory lift, and is relatively well understood.

- Worldsheet CFT.
- F/M-theory.
- Holographic picture.
- Field theory.

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Everything but the CFT approach potentially generalizes to $k = 3, 4, 6$.

Worldsheet description of the O3 plane

We start with IIB string theory on $\mathbb{R}^{10} = \mathbb{R}^4 \times \mathbb{C}^3$, and quotient by $\mathcal{I}(-1)^{F_L}\Omega$. Here $\mathcal{I}$ acts as reflection on the $\mathbb{C}^3$:

$$\mathcal{I}\colon (x, y, z) \to (-x, -y, -z) \tag{5}$$

while $(-1)^{F_L}\Omega$ acts on the worlsheet. Its induced effect on the spacetime fields is easily computed, for instance

$$(-1)^{F_L}\Omega\colon \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ -C_2 \end{pmatrix} \tag{6}$$

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If we have a stack of N D3 branes we need to choose an action on the Chan-Paton factors, which will project $U(N)$ down to an orthogonal or symplectic group:

$\mathrm{O3}^-$ $\quad$ $\widetilde{\mathrm{O3}}^-$ $\quad$ $\mathrm{O3}^+$ $\quad$ $\widetilde{\mathrm{O3}}^+$

Last three are related by Montonen-Olive duality. [Witten '98])

F(M)-theory description of the O3 plane

IIB without orientifold is given by M-theory on T^2 in the $\mathrm{vol}(T^2) \to 0$ limit, we wish to quotient this by the lift of $\mathcal{I}(-1)^{F_L}\Omega$.

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The $\mathcal{I}$ action on the IIB coordinates lifts trivially to a $\mathcal{I}$ action on six of the M-theory coordinates: $(x, y, z) \to (-x, -y, -z)$.

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which when rewritten in terms of C_3 (which is invariant under the lift of $(-1)^{F_L}\Omega$) implies that

$$(-1)^{F_L}\Omega\colon (p, q) \to (-p, -q) \tag{8}$$

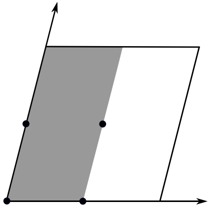
i.e. an inversion of the T^2: $u \to -u$. (Denoted by $-1 \in SL(2, \mathbb{Z})$)

F(M)-theory description of the O3 plane

Writing x, y, z, u for the $\mathbb{C}^3 \times T^2$ coordinates acted upon by the involution, we thus find

$$\mathcal{I}(-1)^{F_L}\Omega \colon (x, y, z, u) \to (-x, -y, -z, -u)$$

and the total geometry is $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$. This has four fixed points at $(x, y, z, u) = (0, 0, 0, p)$, with p a fixed point of the T^2 under the $\mathbb{Z}_2$.



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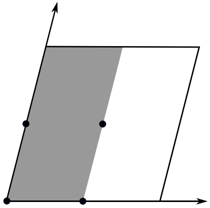

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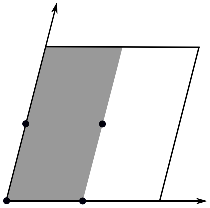

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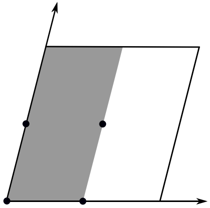

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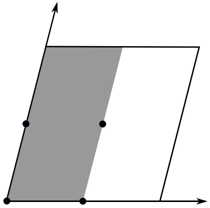

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- Different O3 types: different discrete fluxes on the fixed points [Hanany, Kol '00].

F(IIB)-theory description of the O3 plane

A holography appetizer

In IIB string theory the $\mathbb{C}^3/\mathcal{I}$ orbifold is non-supersymmetric, while the O3 preserves 16 supercharges. I discuss the near horizon geometry, $AdS_5 \times (S^5/\mathbb{Z}_2)$, which naively is non-supersymmetric.

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From the M-theory picture, it is clear what is going on: near horizon what we have is F-theory on $AdS_5 \times ((S^5 \times T^2)/\mathbb{Z}_2)$, i.e. a non-trivial $SL(2,\mathbb{Z})$ bundle on the $S^5/\mathbb{Z}_2$ horizon.

So we do not have the vanilla orbifold, but in addition it has a non-trivial flat $SL(2,\mathbb{Z})$ duality bundle on top, acting with $-1 \in SL(2,\mathbb{Z})$ as we go round the non-trivial one-cycle in the $S^5/\mathbb{Z}_2$ horizon manifold. One can check that the $-1 \in SL(2,\mathbb{Z})$ acting on the sugra spinors restores susy as expected.

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The different kinds of orientifolds in this language are classified by discrete flux: $[H_3], [F_3] \in H^3(S^5/\mathbb{Z}_2, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2$. [Witten '98]

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Similarly, IIB $SL(2,\mathbb{Z})$ descends straightforwardly to the $SL(2,\mathbb{Z})$ duality group of the field theory. In particular

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So we can understand the orientifold projection as a quotient by a particular symmetry of $\mathcal{N} = 4$ $U(N)$ SYM: $U(N)/(\mathbb{Z}_2^R \cdot \mathbb{Z}_2^{SL(2,\mathbb{Z})})$. (In this language we also have a choice of Chan-Paton factors.)

Recap and strategy

We have discussed four ways of viewing the action of an O3 plane on a stack of D3 branes:

- Worldsheet CFT: a projection of the CFT by $\mathcal{I}(-1)^{F_L}\Omega$, with a choice of Chan-Paton factors.
- M-theory: M2 branes probing $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$, with a choice of discrete torsion on the fixed points.
- IIB holography: An orbifold $AdS_5 \times (S^5/\mathcal{I})$ with a nontrivial flat $SL(2,\mathbb{Z})$ bundle, and choice of discrete $[F],[H]$ flux.
- Field theory: A quotient of $U(N)$ SYM by $(\mathbb{Z}_2^R \cdot \mathbb{Z}_2^{SL(2,\mathbb{Z})})$, with a choice of Chan-Paton factors.

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- IIB holography: An orbifold $AdS_5 \times (S^5/\mathcal{I})$ with a nontrivial flat $SL(2,\mathbb{Z})$ bundle, and choice of discrete $[F], [H]$ flux.
- Field theory: A quotient of $U(N)$ SYM by $(\mathbb{Z}_2^R \cdot \mathbb{Z}_2^{SL(2,\mathbb{Z})})$, with a choice of Chan-Paton factors.

Strategy for generalization

Quotient by other possible symmetries of $\mathbb{C}^3 \times T^2$, S^5 or $U(N)$.

The generalization of the CFT approach seems less obvious.

OF3 planes from M-theory

We start by considering the M-theory picture, given by $\mathbb{Z}_k$ $(k > 2)$ quotients of $\mathbb{C}^3 \times T^2$ leaving isolated fixed points. It turns out that maximal supersymmetry ($\mathcal{N} = 3$) is preserved only for $k = 3, 4, 6$, with action [Font, López '04]

$$(x, y, z, u) \to (\omega_k x, \omega_k^{-1} y, \omega_k z, \omega_k^{-1} u) \tag{10}$$

with $\omega_k = \exp(2\pi i/k)$. (These are known to be terminal Gorenstein [Morrison, Stevens '84].) We focus on these.

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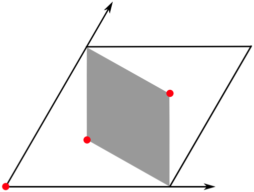
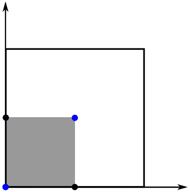
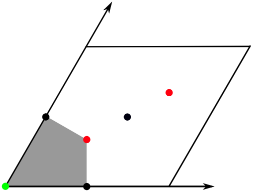
This action only maps the torus to itself for specific complex structures:



$\mathbb{Z}_3$: $\tau = e^{2\pi i/3}$
Three $\mathbb{C}/\mathbb{Z}_3$ points.



$\mathbb{Z}_4$: $\tau = i$
One $\mathbb{Z}_2$ and two $\mathbb{Z}_4$ points.



$\mathbb{Z}_6$: $\tau = e^{2\pi i/3}$
One $\mathbb{Z}_6$, one $\mathbb{Z}_2$ and one $\mathbb{Z}_3$ point.

Holographic perspective

There seems to be no obstruction to taking the F-theory limit, so we end up with a IIB background of the form $\mathbb{C}^3/\mathbb{Z}_k$. Putting D3 branes on the singularity, and taking the near horizon limit, this suggests a dual description for the field theories in terms of $AdS_5 \times (S^5/\mathbb{Z}_k)$, with a non-trivial flat $SL(2,\mathbb{Z})$ bundle. (Provides a microscopic realization of the setup proposed in [Ferrara,Porrati,Zaffaroni '98].)

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Remarkably, the axio-dilaton τ is frozen to a $\mathcal{O}(1)$ value in these backgrounds. We learn that the theories on the branes no longer have the marginal deformation associated to changing the Yang-Mills coupling.

$\mathcal{N} = 4$ quotient perspective

In terms purely of the theory on the probe branes, we start from the observation that for particular (self-dual) values of τ_{YM}, certain $\mathbb{Z}_k$ subgroups of the $SL(2, \mathbb{Z})$ become symmetries. For instance, when $\tau = i$ we have that S-duality

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{11}$$

becomes a symmetry of the theory. $(-i^{-1} = i.)$

We can then construct appropriate quotients

$$\mathcal{Q}_k = \frac{\mathcal{N} = 4 \ U(N)}{\mathbb{Z}_k^R \cdot \mathbb{Z}_k^{SL(2,\mathbb{Z})}} \,. \tag{12}$$

We choose $\mathbb{Z}_k^R$ to be the R-symmetry generator associated with the $\mathbb{Z}_k$ rotation in the transverse $\mathbb{R}^6$, in order to preserve susy.

Supersymmetry

We claim that these theories preserve (just) 12 supercharges for $n > 2$. We now show this in the $\mathcal{N} = 4$ SYM quotient perspective (the computation from the other viewpoints is essentially isomorphic).

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The 16 supercharges arrange into four spacetime spinors Q_α^A, a spinor of $SU(4)_R$. Under the $\mathbb{Z}_k$ rotation these transform as ($\omega_k = \exp(2\pi i/k)$)

$$(Q^1, Q^2, Q^3, Q^4) \to (\omega_k^{\frac{1}{2}} Q^1, \omega_k^{\frac{1}{2}} Q^2, \omega_k^{\frac{1}{2}} Q^3, \omega_k^{-\frac{3}{2}} Q^4)\,. \qquad (13)$$

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The transformation of the supercharge generators under a $SL(2,\mathbb{Z})$ transformation is [Kapustin, Witten '06]

$$Q^A \to \gamma^{\frac{1}{2}} Q^A \qquad \text{with} \quad \gamma = \frac{|c\tau + d|}{c\tau + d} . \qquad (14)$$

For the theories we are constructing, we have $\gamma = \omega_k^{-1}$, so only Q^A with $A = 1, 2, 3$ survive the quotient. (For $\mathbb{Z}_4$: $g_{SL(2,\mathbb{Z})} = S$, $\tau = i$, so $\gamma = -i$, while $\omega_4 = i$.) (Notice that for $k = 1, 2$ we preserve $\mathcal{N} = 4$.)

Properties of the SCFT

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Properties of the SCFT

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During the last few months a beautiful set of results have appeared which (among other things) shed light on the behavior of $\mathcal{N} = 3$ SCFTs in 4d. [Aharony, Evtikhiev '15], [Nishinaka, Tachikawa '16], [Córdova, Dumitrescu, Intriligator '16], [Argyres, Lotito, Lü, Martone '16], [Aharony, Tachikawa '16], [Imamura, Yokoyama '16], [Imamura, Kato, Yokoyama '16], [Agarwal, Amariti '16].

I'll give a very brief summary of what these works say about $\mathcal{N} = 3$ theories.

Relevant and marginal deformations

In [Aharony, Evtikhiev '15] and [Córdova,Dumitrescu,Intriligator '16] it is shown that truly $\mathcal{N}=3$ theories cannot have marginal or relevant deformations preserving $\mathcal{N}=3$.

(Seems to be in good agreement with our construction: $\mathcal{N}=4$ theories have no relevant deformations preserving $\mathcal{N}=4$, and just one marginal deformation preserving $\mathcal{N}=4$: the coupling, which we project out in our quotient.)

Rank one $\mathcal{N} = 3$ theories

It was shown in [Nishinaka, Tachikawa '16] that for rank one $\mathcal{N} = 3$ theories, the form of the moduli space is necessarily $\mathbb{C}^3/\mathbb{Z}_\ell$, with $\ell \in \{1, 2, 3, 4, 6\}$. Furthermore, for $\ell = 1, 2$ one has enhancement to $\mathcal{N} = 4$, while for $\ell = 3, 4, 6$ the theory is purely $\mathcal{N} = 3$.

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The central charge has been computed: $a = c = (2\ell - 1)/4$. (For $\mathcal{N} = 3$ it is always the case that $a = c$. [Aharony, Evtikhiev '15]) (The general form of $a = c$ has been conjectured in [Aharony, Tachikawa '16].)

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$\mathcal{N} = 3$ theories are necessarily $\mathcal{N} = 2$. There is a proposed classification of rank-one $\mathcal{N} = 2$ theories by [Argyres, Lotito, Lü, Martone '16]. The possibilities allowed by the classification are very limited, and the $\mathcal{N} = 3$ theories we find seem to fit well in the classification.

Holographic dual

The holographic dual is F-theory on $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_k$. In [Aharony, Tachikawa '16] and [Imamura, Yokoyama '16] it was shown how to understand the different $\mathrm{OF}3_k$ variants in this language, clarifying a subtlety in an analysis by [Witten '98] for O3 planes, and extending it to $\mathrm{OF}3_{k>2}$.

In particular, this viewpoint gives a way of computing the leading and subleading (in N) contribution to the superconformal index of these theories.

Amusingly, at low N the can be accidental enhancement to $\mathcal{N}=4$, and one finds in this way the "holographic duals" of $\mathcal{N}=4$ with gauge algebras $\mathfrak{su}(3)$, $\mathfrak{so}(5)$ and $\mathfrak{g}_2$. [Aharony, Tachikawa '16], [Imamura, Kato, Yokoyama '16], [Agarwal, Amariti '16].

Question

For $\mathcal{N}=4$ we are only missing $\mathfrak{f}_4$ and $\mathfrak{e}_i$ with $i \in \{6,7,8\}$. Can we construct them with D3 branes? Perhaps from an accidental enhancement of $\mathcal{N}=2$?

Conclusions

- We have constructed the first known examples of $\mathcal{N} = 3$ SCFTs.
- We do so by a very natural F-theoretical generalization of the O3 plane, which freezes out the axio-dilaton, giving intrinsically strongly coupled backgrounds.
- The geometry involves rigid (neither deformable nor resolvable in a Calabi-Yau way) singularities.
- F-theoretical example of branes at singularities.
- The SCFTs we find have natural holographic descriptions as $AdS_5 \times X$, where X is a non-trivial smooth F-theory background with frozen axio-dilaton, realizing the proposal in [Ferrara, Porrati, Zaffaroni '98].
- From F/M duality we have that upon compactification on a circle we flow to $\mathcal{N} \geq 6$ ABJM theories.

Open questions

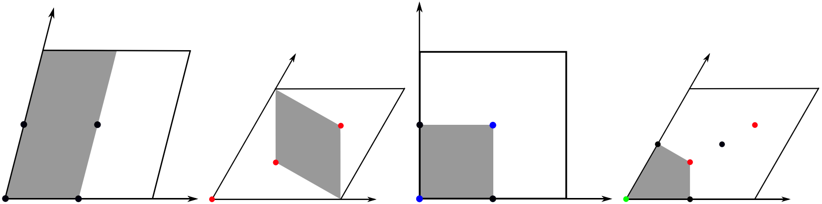
Many!

- Make concrete the notion of Chan-Patons in the $\frac{U(N)\,\mathcal{N}=4}{\text{symmetry}}$ description. BPS states? SCI?
- Other $\mathcal{N}=4$ starting points, beyond $U(N)$?
- Relating the SCI of the $\mathcal{N}=3$ theories to $\mathcal{N}=6$ ABJM partition functions.

Additional Material

Potential OF3 planes

From the M-theory perspective we can classify all possible D3 charges for OF3 planes.



$\mathbb{Z}_2$: τ arb. $\mathbb{Z}_2^{\oplus 4}$ | $\mathbb{Z}_3$: $\tau = e^{2\pi i/3}$ $\mathbb{Z}_3^{\oplus 3}$ | $\mathbb{Z}_4$: $\tau = i$ $\mathbb{Z}_2 \oplus \mathbb{Z}_4^{\oplus 2}$ | $\mathbb{Z}_6$: $\tau = e^{2\pi i/3}$ $\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_6$

Around each $\mathbb{C}^4/\mathbb{Z}_k$ fixed point we can turn on a discrete F_4 flux valued in $H^4(S^7/\mathbb{Z}_k, \mathbb{Z}) = \mathbb{Z}_k$.

Potential OF3 planes

From here we can compute the M2 charge around each fixed point. If the torsion is trivial this comes just from curvature [Bergman, Hirano '09]

$$Q(\mathsf{OM}_{k,0}) = -\frac{\chi(\mathbb{C}^4/\mathbb{Z}_k)}{24} = -\frac{1}{24}\left(k - \frac{1}{k}\right) . \tag{15}$$

The contribution from a $p \in H^4(S^7/bZ_k, \mathbb{Z})$ flux gives an additional term [Aharony, Hashimoto, Hirano, Ouyang '09]

$$Q(\mathsf{OM}_{k,p}) = Q(\mathsf{OM}_{k,0}) + \frac{p(k-p)}{2k}. \tag{16}$$

Potential OF3 planes

Orientifold	Charges
OF_2	$-\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$
OF_3	$-\frac{1}{3}, 0, \frac{1}{3}, \frac{2}{3}$
OF_4	$-\frac{3}{8}, -\frac{1}{8}, 0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}$
OF_6	$-\frac{5}{12}, -\frac{1}{6}, -\frac{1}{12}, 0, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{5}{6}, \frac{11}{12}$

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But notice!

Not all of these M-theory settings lift to non-trivial orientifolds in IIB!

Classification results

The proper classification was achieved by [Aharony, Tachikawa '16].

$$H^3(S^5/\mathbb{Z}_k, (\mathbb{Z}\oplus\mathbb{Z})_\rho) = \begin{cases} \mathbb{Z}_2 \oplus \mathbb{Z}_2 & (k=2) \\ \mathbb{Z}_3 & (k=3) \\ \mathbb{Z}_2 & (k=4) \\ \mathbb{Z}_1 & (k=6) \end{cases} \tag{17}$$

or alternatively, directly seeing which fluxes lift in F-theory to non-shift orientifolds.