T-branes through 3d mirror symmetry

Simone Giacomelli

Université Libre de Bruxelles and Solvay Institutes

Bern, July 04 2016

Based on: A. Collinucci, S.G., R. Savelli and R. Valandro
arXiv:1603.00062[hep-th]

Branes and F/M-theory geometry

A stack of N D_p branes supports a $U(N)$ gauge theory and the vev of the scalars Φ_i in the vectormultiplet parametrizes the position of the branes.
In M/F-theory these data (eigenvalues of Φ_i) are encoded in the geometric properties of the background.

In the case of D7 branes we have the BPS equation $[\Phi, \Phi^\dagger] \sim F_A$ and if we turn on the gauge flux we can consider a non diagonalizable Higgs field! S. Cecotti, C. Cordova, J. Heckman, C. Vafa '10.

A brane configuration with nilpotent Φ is called **T-brane**!

Branes and F/M-theory geometry

A stack of N D_p branes supports a $U(N)$ gauge theory and the vev of the scalars Φ_i in the vectormultiplet parametrizes the position of the branes.
In M/F-theory these data (eigenvalues of Φ_i) are encoded in the geometric properties of the background.

In the case of D7 branes we have the BPS equation $[\Phi, \Phi^\dagger] \sim F_A$ and if we turn on the gauge flux we can consider a non diagonalizable Higgs field! S. Cecotti, C. Cordova, J. Heckman, C. Vafa '10.

A brane configuration with nilpotent Φ is called **T-brane**!

Branes and F/M-theory geometry

A stack of N D_p branes supports a $U(N)$ gauge theory and the vev of the scalars Φ_i in the vectormultiplet parametrizes the position of the branes.
In M/F-theory these data (eigenvalues of Φ_i) are encoded in the geometric properties of the background.

In the case of D7 branes we have the BPS equation $[\Phi, \Phi^{\dagger}] \sim F_A$ and if we turn on the gauge flux we can consider a non diagonalizable Higgs field! S. Cecotti, C. Cordova, J. Heckman, C. Vafa '10.

A brane configuration with nilpotent Φ is called **T-brane**!

T-branes in M-theory and probes

One way to characterize compactifications of F-theory is in terms of a dual description in M-theory:

$$\text{M-theory on X} \sim \text{F-theory on } S^1 \times X.$$

On a stack of D6 branes there are three scalars Φ_i. A T-brane is defined by $[\langle\Phi_i\rangle, \langle\Phi_j\rangle] \neq 0$. We consider the case of nilpotent vev for $\Phi_{D6} = \Phi_1 + i\Phi_2$.

Since we don't have a definition of T-brane in M-theory, we consider the 3d theory on a 2-brane probing a T-brane background. For simplicity we will restrict to ADE singularities.

T-branes in M-theory and probes

One way to characterize compactifications of F-theory is in terms of a dual description in M-theory:

$$\text{M-theory on X} \sim \text{F-theory on } S^1 \times X.$$

On a stack of D6 branes there are three scalars Φ_i. A T-brane is defined by $[\langle \Phi_i \rangle, \langle \Phi_j \rangle] \neq 0$. We consider the case of nilpotent vev for $\Phi_{D6} = \Phi_1 + i\Phi_2$.

Since we don't have a definition of T-brane in M-theory, we consider the 3d theory on a 2-brane probing a T-brane background. For simplicity we will restrict to ADE singularities.

T-branes in M-theory and probes

One way to characterize compactifications of F-theory is in terms of a dual description in M-theory:

$$\text{M-theory on X} \sim \text{F-theory on } S^1 \times X.$$

On a stack of D6 branes there are three scalars Φ_i. A T-brane is defined by $[\langle \Phi_i \rangle, \langle \Phi_j \rangle] \neq 0$. We consider the case of nilpotent vev for $\Phi_{D6} = \Phi_1 + i\Phi_2$.

Since we don't have a definition of T-brane in M-theory, we consider the 3d theory on a 2-brane probing a T-brane background. For simplicity we will restrict to ADE singularities.

$\mathcal{N} = 4$ moduli space and mirror symmetry

- Vectormultiplet: (A_μ, σ, Φ).
- Hypermultiplet: (Φ_1, Φ_2).
- Monopole operators: $d\gamma = *dA$, $W_\pm = e^{\sigma \pm i\gamma}$

Coulomb branch: space of vacua where only vectormultiplet scalars and monopoles have a vev. It is modified by quantum corrections.

Higgs Branch: space of vacua where only hypermultiplet scalars have a vev. Unaffected by quantum corrections.

Mirror Symmetry (K. Intriligator, N. Seiberg '96)

Duality between $\mathcal{N} = 4$ theories exchanging Coulomb and Higgs branches.

$\mathcal{N} = 4$ moduli space and mirror symmetry

- Vectormultiplet: (A_μ, σ, Φ).
- Hypermultiplet: (Φ_1, Φ_2).
- Monopole operators: $d\gamma = *dA$, $W_\pm = e^{\sigma \pm i\gamma}$

Coulomb branch: space of vacua where only vectormultiplet scalars and monopoles have a vev. It is modified by quantum corrections.

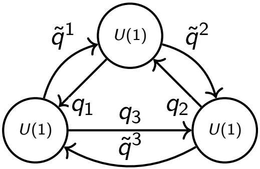
Higgs Branch: space of vacua where only hypermultiplet scalars have a vev. Unaffected by quantum corrections.

Mirror Symmetry (K. Intriligator, N. Seiberg '96)

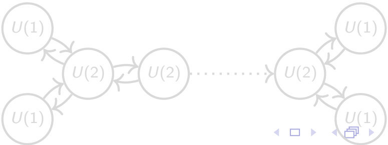
Duality between $\mathcal{N} = 4$ theories exchanging Coulomb and Higgs branches.

D6 branes VS abelian singularity

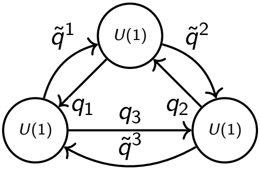
Theory A: (D2 on top of N D6 branes) SQED with N flavors.
Theory B: (D2 at a A_{N-1} singularity) circular quiver with N gauge groups $(\mathcal{W} = \sum_i S_i q_i \tilde{q}^i - \Psi \sum_i S_i)$.



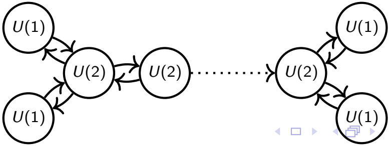
Theory A: (D2 on top of N D6 and O6 plane) $SU(2)$ SQCD with N flavors.
Theory B: (D2 brane probing a singularity of type D_N) unitary quiver with affine D_N shape.



Theory A: (D2 on top of N D6 branes) SQED with N flavors.
Theory B: (D2 at a A_{N-1} singularity) circular quiver with N gauge groups $(\mathcal{W} = \sum_i S_i q_i \tilde{q}^i - \Psi \sum_i S_i)$.



Theory A: (D2 on top of N D6 and O6 plane) $SU(2)$ SQCD with N flavors.
Theory B: (D2 brane probing a singularity of type D_N) unitary quiver with affine D_N shape.



Coulomb vs Higgs branches and T-branes

In Theory A monopole operators satisfy V. Borokhov, A. Kapustin, X. Wu '02.

$$W_+ W_- \sim \Phi^N \quad (A_{N-1} \text{ singularity})$$

In Theory B we have $\mathcal{W} = \sum_i S_i q_i \tilde{q}^i - \Psi \sum_i S_i$

$$0 = \partial \mathcal{W} / \partial S_i = q_i \tilde{q}^i - \Psi.$$

$$B\tilde{B} = \prod_i q_i \tilde{q}^i = \Psi^N \quad (B = \prod_i q_i, \; \tilde{B} = \prod_i \tilde{q}^i).$$

In the $D2$ theory, $\langle \Phi_{D6} \rangle$ is interpreted as the mass $m_i^j Q_j \tilde{Q}^i$.

$$\Downarrow$$

Using the mirror map, a T-brane deforms theory B by

$$\delta \mathcal{W} = m W_{i,+}.$$

Coulomb vs Higgs branches and T-branes

In Theory A monopole operators satisfy V. Borokhov, A. Kapustin, X. Wu '02.

$$W_+ W_- \sim \Phi^N \quad (A_{N-1} \text{ singularity})$$

In Theory B we have $\mathcal{W} = \sum_i S_i q_i \tilde{q}^i - \Psi \sum_i S_i$

$$0 = \partial \mathcal{W} / \partial S_i = q_i \tilde{q}^i - \Psi.$$

$$B\tilde{B} = \prod_i q_i \tilde{q}^i = \Psi^N \quad (B = \prod_i q_i, \ \tilde{B} = \prod_i \tilde{q}^i).$$

In the $D2$ theory, $\langle \Phi_{D6} \rangle$ is interpreted as the mass $m_i^j Q_j \tilde{Q}^i$.

$$\Downarrow$$

Using the mirror map, a T-brane deforms theory B by

$$\delta \mathcal{W} = m W_{i,+}.$$

Coulomb vs Higgs branches and T-branes

In Theory A monopole operators satisfy V. Borokhov, A. Kapustin, X. Wu '02.

$$W_+ W_- \sim \Phi^N \quad (A_{N-1} \text{ singularity})$$

In Theory B we have $\mathcal{W} = \sum_i S_i q_i \tilde{q}^i - \Psi \sum_i S_i$

$$0 = \partial \mathcal{W} / \partial S_i = q_i \tilde{q}^i - \Psi.$$

$$B\tilde{B} = \prod_i q_i \tilde{q}^i = \Psi^N \quad (B = \prod_i q_i, \ \tilde{B} = \prod_i \tilde{q}^i).$$

In the $D2$ theory, $\langle \Phi_{D6} \rangle$ is interpreted as the mass $m_i^j Q_j \tilde{Q}^i$.

$$\Downarrow$$

Using the mirror map, a T-brane deforms theory B by

$$\delta \mathcal{W} = m W_{i,+}.$$

Coulomb vs Higgs branches and T-branes

In Theory A monopole operators satisfy V. Borokhov, A. Kapustin, X. Wu '02.

$$W_+ W_- \sim \Phi^N \quad (A_{N-1} \text{ singularity})$$

In Theory B we have $\mathcal{W} = \sum_i S_i q_i \tilde{q}^i - \Psi \sum_i S_i$

$$0 = \partial \mathcal{W} / \partial S_i = q_i \tilde{q}^i - \Psi.$$

$$B\tilde{B} = \prod_i q_i \tilde{q}^i = \Psi^N \quad (B = \prod_i q_i, \ \tilde{B} = \prod_i \tilde{q}^i).$$

In the $D2$ theory, $\langle \Phi_{D6} \rangle$ is interpreted as the mass $m_i^j Q_j \tilde{Q}^i$.

$$\Downarrow$$

Using the mirror map, a T-brane deforms theory B by

$$\delta \mathcal{W} = m W_{i,+}.$$

$\mathcal{N} = 2$ abelian mirror symmetry

The mirror for $\mathcal{N} = 2$ abelian theories is known O. Aharony et al. '97

Theory A: $\mathcal{N} = 2$ SQED with N flavors ($\mathcal{W} = 0$)

Theory B: quiver with N gauge groups ($\mathcal{W} = \sum_i S_i q_i \tilde{q}^i$)

- For $N = 2$: Theory A is SQED with 2 flavors.
  Theory B is Theory A with $\mathcal{W} = S_1 q_1 \tilde{q}^1 + S_2 q_2 \tilde{q}^2$.
- For $N = 1$: Theory A is SQED with one flavor.
  Theory B describes 3 chirals with $\mathcal{W} = XYZ$.
  $X \leftrightarrow Q\tilde{Q}, \quad Y \leftrightarrow W_+, \quad Z \leftrightarrow W_-$.

Local mirror symmetry

All nodes are $U(N)$ theories with $2N$ flavors so, under mirror symmetry monopoles are mapped to mass terms.

- Make the gauge coupling at neighbouring nodes small
- Consider the mirror of the gauge node "in isolation" and integrate out massive fields
- Extract the mirror of the resulting theory and couple it to the rest of the quiver

In the A_{N-1} case we have a $U(1)$ theory with 2 flavors

$$\mathcal{W} = S_1 q_1 \tilde{q}^1 + S_2 q_2 \tilde{q}^2 + mW_+ - \Psi(S_1 + S_2) + \ldots$$

Integrating out the massive flavor in the mirror side we find

$$\mathcal{W} = -\Psi'^2 Q\tilde{Q}/m.$$

Its mirror is the XYZ model with

$$\mathcal{W} = -\Psi^2 X/m + XYZ.$$

Local mirror symmetry

All nodes are $U(N)$ theories with $2N$ flavors so, under mirror symmetry monopoles are mapped to mass terms.

- Make the gauge coupling at neighbouring nodes small
- Consider the mirror of the gauge node "in isolation" and integrate out massive fields
- Extract the mirror of the resulting theory and couple it to the rest of the quiver

In the A_{N-1} case we have a $U(1)$ theory with 2 flavors

$$\mathcal{W} = S_1 q_1 \tilde{q}^1 + S_2 q_2 \tilde{q}^2 + mW_+ - \Psi(S_1 + S_2) + \dots$$

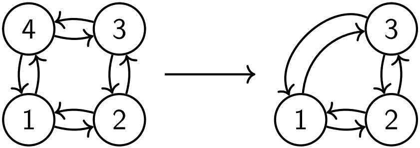
Integrating out the massive flavor in the mirror side we find

$$\mathcal{W} = -\Psi'^2 Q\tilde{Q}/m.$$

Its mirror is the XYZ model with

$$\mathcal{W} = -\Psi^2 X/m + XYZ.$$

Resolutions and deformations



In the $\mathcal{N}=4$ theory with N flavors $\mathcal{W}=\sum_{i=1}^{N} S_i(q_i\tilde{q}^i-\Psi)$:

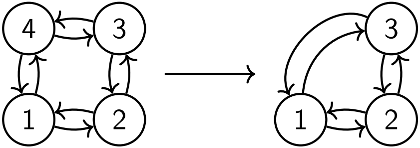
- $N-1$ deformation parameters: $\delta\mathcal{W}=\lambda_i S_i$.
- N resolution parameters: FI terms $\int d^4\theta\xi_i V_i$.

Turning on a T-brane $\mathcal{W}=\sum_{i=1}^{N-2} S_i(q_i\tilde{q}^i-\Psi)+S(q\tilde{q}-\Psi^2)$:

- $N-1$ deformation parameters: $\delta\mathcal{W}=\lambda_i S_i$.
- $N-1$ resolution parameters: FI terms $\int d^4\theta\xi_i V_i$.

The Higgs branch is the same as in the parent $\mathcal{N}=4$ theory but the resolution of the singularity is obstructed!

Resolutions and deformations



In the $\mathcal{N}=4$ theory with N flavors $\mathcal{W}=\sum_{i=1}^{N} S_i(q_i\tilde{q}^i-\Psi)$:

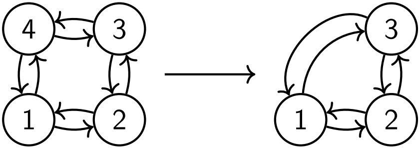
- $N-1$ deformation parameters: $\delta\mathcal{W}=\lambda_i S_i$.
- N resolution parameters: FI terms $\int d^4\theta\xi_i V_i$.

Turning on a T-brane $\mathcal{W}=\sum_{i=1}^{N-2} S_i(q_i\tilde{q}^i-\Psi)+S(q\tilde{q}-\Psi^2)$:

- $N-1$ deformation parameters: $\delta\mathcal{W}=\lambda_i S_i$.
- $N-1$ resolution parameters: FI terms $\int d^4\theta\xi_i V_i$.

The Higgs branch is the same as in the parent $\mathcal{N}=4$ theory but the resolution of the singularity is obstructed!

Resolutions and deformations



In the $\mathcal{N}=4$ theory with N flavors $\mathcal{W}=\sum_{i=1}^{N} S_i(q_i\tilde{q}^i-\Psi)$:

- $N-1$ deformation parameters: $\delta\mathcal{W}=\lambda_i S_i$.
- N resolution parameters: FI terms $\int d^4\theta \xi_i V_i$.

Turning on a T-brane $\mathcal{W}=\sum_{i=1}^{N-2} S_i(q_i\tilde{q}^i-\Psi)+S(q\tilde{q}-\Psi^2)$:

- $N-1$ deformation parameters: $\delta\mathcal{W}=\lambda_i S_i$.
- $N-1$ resolution parameters: FI terms $\int d^4\theta \xi_i V_i$.

The Higgs branch is the same as in the parent $\mathcal{N}=4$ theory but the resolution of the singularity is obstructed!

T-branes for the D_N theory

At an abelian node we have the superpotential

$$\mathcal{W} = -\phi \,\mathrm{Tr}(q\tilde{q}) + \mathrm{Tr}(\Phi_{U(2)} q\tilde{q}) + W_+$$

Integrating out the massive flavor in the mirror

$$\mathcal{W} = -\phi(S_1 + S_2) + \mathrm{Tr}(\Phi_{U(2)} M) - S_1 S_2 Q\tilde{Q}$$

and mirroring again

$$\mathcal{W} = -\phi \,\mathrm{Tr}\, M + \mathrm{Tr}(\Phi_{U(2)} M) - X \det M; \;\; M = \begin{pmatrix} S_1 & Y \\ Z & S_2 \end{pmatrix}$$

The D_N singularity is preserved, the blow-up is obstructed!

T-branes for the D_N theory

At an abelian node we have the superpotential

$$\mathcal{W} = -\phi\,\mathrm{Tr}(q\tilde{q}) + \mathrm{Tr}(\Phi_{U(2)}q\tilde{q}) + W_+$$

Integrating out the massive flavor in the mirror

$$\mathcal{W} = -\phi(S_1 + S_2) + \mathrm{Tr}(\Phi_{U(2)}M) - S_1 S_2 Q\tilde{Q}$$

and mirroring again

$$\mathcal{W} = -\phi\,\mathrm{Tr}\,M + \mathrm{Tr}(\Phi_{U(2)}M) - X\det M;\;\; M = \begin{pmatrix} S_1 & Y \\ Z & S_2 \end{pmatrix}$$

The D_N singularity is preserved, the blow-up is obstructed!

T-branes for the D_N theory

T-branes through 3d mirror symmetry

Simone Giacomelli

T-branes in string theory

3D Supersymmetry

Monopoles and mirror symmetry

At an abelian node we have the superpotential

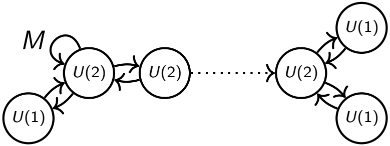
$$\mathcal{W} = -\phi\,\mathrm{Tr}(q\tilde{q}) + \mathrm{Tr}(\Phi_{U(2)} q\tilde{q}) + W_+$$

Integrating out the massive flavor in the mirror

$$\mathcal{W} = -\phi(S_1 + S_2) + \mathrm{Tr}(\Phi_{U(2)} M) - S_1 S_2 Q \tilde{Q}$$

and mirroring again

$$\mathcal{W} = -\phi\,\mathrm{Tr}\,M + \mathrm{Tr}(\Phi_{U(2)} M) - X \det M; \;\; M = \begin{pmatrix} S_1 & Y \\ Z & S_2 \end{pmatrix}$$



The D_N singularity is preserved, the blow-up is obstructed!

Concluding remarks

We proposed a method to understand the properties of T-branes through the wordvolume theory of a brane probing the geometry.

We found a quiver gauge theory description telling us that T-branes do not deform the geometry but obstruct resolutions!

For D, E singularities we can understand the case of minimal nilpotent mass matrices. The general case requires knowledge of nonabelian $\mathcal{N} = 2$ mirror symmetry.

It would be interesting to apply this method to more complicated backgrounds/brane systems.

Thank You!

Concluding remarks

We proposed a method to understand the properties of T-branes through the wordvolume theory of a brane probing the geometry.
We found a quiver gauge theory description telling us that T-branes do not deform the geometry but obstruct resolutions!

For D, E singularities we can understand the case of minimal nilpotent mass matrices. The general case requires knowledge of nonabelian $\mathcal{N} = 2$ mirror symmetry.
It would be interesting to apply this method to more complicated backgrounds/brane systems.

Thank You!

Concluding remarks

We proposed a method to understand the properties of T-branes through the wordvolume theory of a brane probing the geometry.
We found a quiver gauge theory description telling us that T-branes do not deform the geometry but obstruct resolutions!

For D, E singularities we can understand the case of minimal nilpotent mass matrices. The general case requires knowledge of nonabelian $\mathcal{N} = 2$ mirror symmetry.
It would be interesting to apply this method to more complicated backgrounds/brane systems.

Thank You!