AdS_4 Black Holes and 3d Gauge Theories

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Supersymmetric theories, dualities and deformations
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F. Benini-AZ; arXiv 1504.03698 and 1605.06120

F. Benini-K.Hristov-AZ; arXiv 1511.04085 and to appear

S. M. Hosseini-AZ; arXiv 1604.03122

Introduction

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- the entropy of a supersymmetric AdS_4 black hole in M theory
- a field theory computation for a partition function in the dual CFT_3

The computation uses recent localization techniques that allow to evaluate exact quantities in supersymmetric gauge theories.

Introduction

One of the success of string theory is the microscopic counting of micro-states for a class of asymptotically flat black holes entropy [Vafa-Strominger'96]

- The black holes are realized by putting together D-branes, extended objects that have gauge theories on the world-volume
- The entropy is obtained by counting states in the corresponding gauge theory

Introduction

No similar result for AdS black holes in $d \geq 4$. But AdS should be simpler and related to holography:

- A gravity theory in AdS_{d+1} is the dual description of a CFT_d

The entropy should be related to the counting of states in the dual CFT. People tried hard for AdS_5 black holes (states in N=4 SYM). Still an open problem.

Prelude

Objects of interest

AdS$_4$ black holes

The objects of interest are BPS asymptotically AdS$_4$ static black holes

$$\mathrm{d}s^2 = e^{\mathcal{K}(X)}\left(gr + \frac{c}{2gr}\right)^2 \mathrm{d}t^2 - \frac{e^{-\mathcal{K}(X)}\mathrm{d}r^2}{\left(gr + \frac{c}{2gr}\right)^2} - e^{-\mathcal{K}(X)} r^2 \mathrm{d}s^2_{\Sigma_g}$$

- supported by magnetic charges on Σ_g: $\mathfrak{n} = \frac{1}{2\pi}\int_{\Sigma_g^2} F$
- preserving supersymmetry via an R-symmetry twist

$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu \epsilon \qquad \Longrightarrow \qquad \epsilon = \text{cost}$$

[Cacciatori,Klemm; Gnecchi,Dall'agata; Hristov,Vandoren;Halmagyi;Katmadas]

Holographic Perspective

General vacua of a bulk effective action

$$\mathcal{L} = -\frac{1}{2}\mathcal{R} + F_{\mu\nu}F^{\mu\nu} + V \dots$$

with a metric

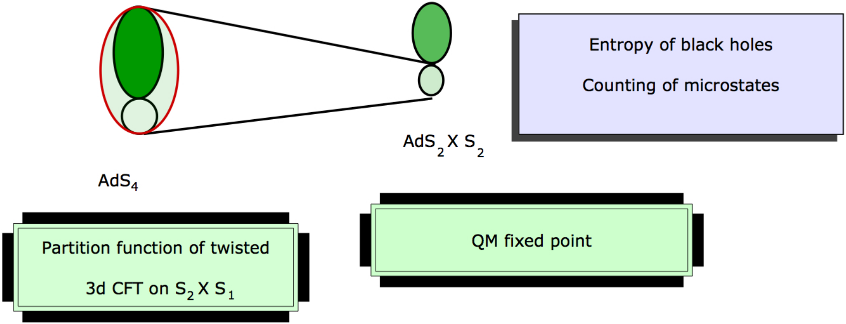
$$ds^2_{d+1} = \frac{dr^2}{r^2} + (r^2 ds^2_{M_d} + O(r)) \qquad A = A_{M_d} + O(1/r)$$

and a gauge fields profile, correspond to CFTs on a d-manifold M_d and a non trivial background field for the R- or global symmetry

$$L_{CFT} + J^\mu A_\mu$$

AdS_4 black holes and holography

AdS black holes are dual to a topologically twisted CFT on $S^2 \times S^1$



[In one dimension more: Benini-Bobev]

Part I

The index for topologically twisted theories in 3d

The topological twist

Consider an $\mathcal{N} = 2$ gauge theory on $S^2 \times S^1$

$$ds^2 = R^2\big(d\theta^2 + \sin^2\theta\, d\varphi^2\big) + \beta^2 dt^2$$

with a magnetic background for the R- and flavor symmetries:

$$A^R = -\frac{1}{2}\cos\theta\, d\varphi = -\frac{1}{2}\omega^{12}\,, \qquad A^F = -\frac{\mathfrak{n}^F}{2}\cos\theta\, d\varphi = -\frac{\mathfrak{n}^F}{2}\omega^{12}$$

In particular A^R is equal to the spin connection so that

$$D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon - iA_\mu^R\epsilon = 0 \qquad \Longrightarrow \qquad \epsilon = \text{const}$$

This is just a topological twist. [Witten '88]

The background

Supersymmetry can be preserved by turning on supersymmetric backgrounds for the flavor symmetry multiplets (A_μ^F, σ^F, D^F):

$$u^F = A_t^F + i\sigma^F \,, \qquad\qquad \mathfrak{n}^F = \int_{S^2} F^F = iD^F$$

and the path integral, which can be exactly computed by localization, becomes a function of a set of magnetic charges $\mathfrak{n}^F$ and chemical potentials u^F.

[Benini-AZ; arXiv 1504.03698]

A topologically twisted index

The path integral can be re-interpreted as a twisted index: a trace over the Hilbert space $\mathcal{H}$ of states on a sphere in the presence of a magnetic background for the R and the global symmetries,

$$\mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ_F A^F} e^{-\beta H}\right)$$

$Q^2 = H - \sigma^F J_F$

holomorphic in u^F

where J_F is the generator of the global symmetry.

The partition function

The path integral on $S^2 \times S^1$ reduces as usual, by localization, to a matrix model depending on few zero modes of the gauge multiplet $V = (A_\mu, \sigma, \lambda, \lambda^\dagger, D)$

- A magnetic flux on S^2, $\mathfrak{m} = \frac{1}{2\pi}\int_{S^2} F$ in the co-root lattice
- A Wilson line A_t along S^1
- The vacuum expectation value σ of the real scalar

The path integral reduces to an r-dimensional contour integral of a meromorphic form

$$\boxed{\frac{1}{|W|}\sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}}\oint_C Z_{\text{int}}(u,\mathfrak{m})} \qquad u = A_t + i\sigma$$

The partition function

- In each sector with gauge flux $\mathfrak{m}$ we have a meromorphic form

$$Z_{\text{int}}(u,\mathfrak{m}) = Z_{\text{class}} Z_{\text{1-loop}}$$

$$\boxed{Z^{\text{CS}}_{\text{class}} = x^{k\mathfrak{m}}} \qquad x = e^{iu}$$

$$\boxed{Z^{\text{chiral}}_{\text{1-loop}} = \prod_{\rho\in\mathfrak{R}} \left[\frac{x^{\rho/2}}{1-x^{\rho}}\right]^{\rho(\mathfrak{m})-q+1}} \qquad q = \text{R charge}$$

$$\boxed{Z^{\text{gauge}}_{\text{1-loop}} = \prod_{\alpha\in G} (1-x^{\alpha})\,(i\,du)^r}$$

- Supersymmetric localization selects a particular contour of integration C and picks some of the residues of the form $Z_{\text{int}}(u,\mathfrak{m})$.

[Jeffrey-Kirwan residue - similar to Benini,Eager,Hori,Tachikawa '13; Hori,Kim,Yi '14]

A Simple Example: SQED

The theory has gauge group $U(1)$ and two chiral Q and $\tilde{Q}$

$$Z = \sum_{\mathfrak{m}\in\mathbb{Z}} \int \frac{dx}{2\pi i\, x} \left(\frac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{1-xy}\right)^{\mathfrak{m}+\mathfrak{n}} \left(\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{1-x^{-1}y}\right)^{-\mathfrak{m}+\mathfrak{n}}$$

	$U(1)_g$	$U(1)_A$	$U(1)_R$
Q	1	1	1
$\tilde{Q}$	-1	1	1

Consistent with duality with three chirals with superpotential XYZ

$$Z = \left(\frac{y}{1-y^2}\right)^{2\mathfrak{n}-1} \left(\frac{y^{-\frac{1}{2}}}{1-y^{-1}}\right)^{-\mathfrak{n}+1} \left(\frac{y^{-\frac{1}{2}}}{1-y^{-1}}\right)^{-\mathfrak{n}+1}$$

Dualities and generalizations

Many generalizations

- We can add refinement for angular momentum on S^2.
- We can consider higher genus $S^2 \to \Sigma$ [also Closset-Kim '16]

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We can go up and down in dimension and compute

- amplitudes in gauged linear sigma models for $(2,2)$ theories in 2d on S^2 [also Cremonesi-Closset-Park '15]
- an elliptically generalized twisted index for $\mathcal{N}=1$ theory on $S^2 \times T^2$ [also Closset-Shamir '13;Nishioka-Yaakov '14;Yoshida-Honda '15]

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The index adds to and complete the list of existing tools (superconformal indices, sphere partition functions) for testing dualities: Aharony; Giveon-Kutasov in 3d; Seiberg in 4d,$\cdots$.

Part II

Comparison with the black hole entropy

Going back to black holes

Consider BPS asymptotically AdS$_4$ static dyonic black holes

$$\mathrm{d}s^2 = e^{\mathcal{K}(X)}\left(gr + \frac{c}{2gr}\right)^2 \mathrm{d}t^2 - \frac{e^{-\mathcal{K}(X)}\mathrm{d}r^2}{\left(gr + \frac{c}{2gr}\right)^2} - e^{-\mathcal{K}(X)} r^2 \mathrm{d}s^2_{\Sigma_g}$$

$$X^i = X^i(r)$$

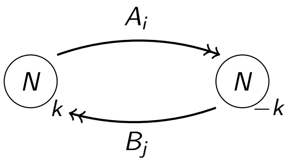
- vacua of $N = 2$ gauged supergravities arising from M theory on AdS$_4 \times S^7$
- electric and magnetic charges for $U(1)^4 \subset SO(8)$
- preserving supersymmetry via an R-symmetry twist

$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu \epsilon \qquad \Longrightarrow \qquad \epsilon = \text{cost}$$

[Cacciatori,Klemm; Gnecchi,Dall'agata; Hristov,Vandoren;Halmagyi;Katmadas]

Going back to black holes

The dual field theory to $AdS_4 \times S^7$ is known: is the ABJM theory with gauge group $U(N) \times U(N)$



with quartic superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

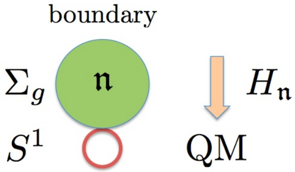
with R and global symmetries

$$U(1)^4 \subset SU(2)_A \times SU(2)_B \times U(1)_B \times U(1)_R \subset SO(8)$$

.

ABJM and the AdS$_4$ black holes

The boundary ABJM theory is topologically twisted, with a magnetic charge for the R-symmetry and for the global symmetries of the theory



$$Z^{\text{twisted}}_{\Sigma_g^2\times S^1}(\mathfrak{n},\Delta) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{n}}}\right)$$

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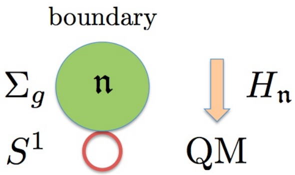

$$Z^{\text{twisted}}_{\Sigma_g^2\times S^1}(\mathfrak{n},\Delta) = \mathrm{Tr}_{\mathcal{H}}\left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{n}}}\right)$$

This is the Witten index of the QM obtained by reducing $\Sigma_g^2 \times S^1 \to S^1$.

- magnetic charges $\mathfrak{n}$ are not vanishing at the boundary and appear in the Hamiltonian
- electric charges can be introduced using chemical potentials Δ

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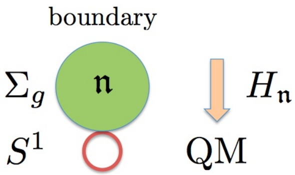


$$Z^{\text{twisted}}_{\Sigma_g^2 \times S^1}(\mathfrak{n}, \Delta) = \mathrm{Tr}_{\mathcal{H}} \left((-1)^F e^{iJ\Delta} e^{-\beta H_{\mathfrak{n}}} \right)$$

The BH entropy is related to a Legendre Transform of the index [Benini-Hristov-AZ]

$$S_{BH}(\mathfrak{q}, \mathfrak{n}) \equiv \mathbb{R}\mathrm{e}\, \mathcal{I}(\Delta) = \mathbb{R}\mathrm{e}(\log Z(\mathfrak{n}, \Delta) - i\Delta\mathfrak{q}) \,, \qquad \frac{d\mathcal{I}}{d\Delta} = 0$$

[similar to Sen's formalism, OSV, etc]

The dual field theory

The ABJM twisted index is

$$Z = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\widetilde{\mathfrak{m}}\in\mathbb{Z}^N} \int \prod_{i=1}^{N} \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k\mathfrak{m}_i} \tilde{x}_i^{-k\widetilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^{N} \left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)\times$$
$$\times \prod_{i,j=1}^{N} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}\,y_1}}{1-\frac{x_i}{\tilde{x}_j}\,y_1}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{n}_1+1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}\,y_2}}{1-\frac{x_i}{\tilde{x}_j}\,y_2}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{n}_2+1}$$
$$\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}\,y_3}}{1-\frac{\tilde{x}_j}{x_i}\,y_3}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{n}_3+1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}\,y_4}}{1-\frac{\tilde{x}_j}{x_i}\,y_4}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{n}_4+1}$$

where $\mathfrak{m},\widetilde{\mathfrak{m}}$ are the gauge magnetic fluxes and y_i are fugacities for the three independent $U(1)$ global symmetries ($\prod_i y_i = 1$)

The dual field theory

Strategy:

- Re-sum geometric series in $\mathfrak{m}, \widetilde{\mathfrak{m}}$.

$$Z = \int \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} \frac{f(x_i, \tilde{x}_i)}{\prod_{j=1}^{N}(e^{iB_i} - 1) \prod_{j=1}^{N}(e^{i\widetilde{B}_j} - 1)}$$

- Step 1: find the zeros of denominator $e^{iB_i} = e^{i\tilde{B}_j} = 1$ at large N

- Step 2: evaluate the residues at large N

$$Z \sim \sum_I \frac{f(x_i^{(0)}, \tilde{x}_i^{(0)})}{\det \mathbb{B}}$$

[Benini-Hristov-AZ; arXiv 1511.04085]

The large N limit

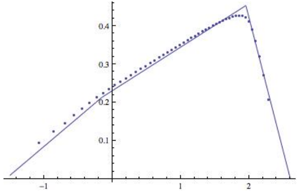
Step 1: solve the large N Limit of algebraic equations giving the positions of poles

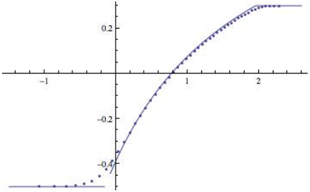
$$1 = x_i^k \prod_{j=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)} = \tilde{x}_j^k \prod_{i=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)}$$

Bethe Ansatz Equations - derived by a potential $\mathcal{V}_{BA}(x_i, \tilde{x}_i)$

with an ansatz

$$\log x_i = i\sqrt{N} t_i + v_i \,, \qquad \log \tilde{x}_i = i\sqrt{N} t_i + \tilde{v}_i$$





The large N limit amusement

- In the large N limit, these *auxiliary* BAE are the same appearing in a different localization problem: the path integral on S^3 [Hosseini-AZ; arXiv 1604.03122]

$$\mathcal{V}_{BA}(\Delta) = Z_{S^3}(\Delta) \qquad\qquad y_i = e^{i\Delta_i}$$

The large N limit amusement

- In the large N limit, these *auxiliary* BAE are the same appearing in a different localization problem: the path integral on S^3 [Hosseini-AZ; arXiv 1604.03122]

$$\mathcal{V}_{BA}(\Delta) = Z_{S^3}(\Delta) \qquad\qquad y_i = e^{i\Delta_i}$$

The same holds for other 3d quivers dual to M theory backgrounds $\mathrm{AdS}_4 \times Y_7$ ($N^{3/2}$) and massive type IIA ones ($N^{5/3}$) [Hosseini-AZ; Hosseini-Mekareeya]

The large N limit

Step 2: plug into the partition function. It is crucial to keep into account exponentially small corrections in tail regions where $y_i x_i/\tilde{x}_i = 1$

$$\log Z = N^{3/2}(\text{finite}) + N\log(1 - y_i x_i/\tilde{x}_i) \qquad y_i x_i/\tilde{x}_i = 1 + e^{-N^{1/2} Y_i}$$

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The result is encoded in a general simple formula [Hosseini-AZ; arXiv 1604.03122]

$$\log Z = -\sum_I \mathfrak{n}_I \frac{\partial \mathcal{V}_{BA}}{\partial \Delta_I}$$

The main result

The index is obtained from $\mathcal{V}_{BA} \sim \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$:

$$\mathcal{I}(\Delta) = \frac{1}{3} N^{3/2} \sum_i \left(-\sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4}\, \frac{\mathfrak{n}_i}{\Delta_i} - i\Delta_i \mathfrak{q}_i \right) \qquad y_i = e^{i\Delta_i}$$

This function can be extremized with respect to the Δ_i and

$$\boxed{\mathbb{R}\mathrm{e}\, \mathcal{I}|_{crit} = \mathrm{BH\, Entropy}(\mathfrak{n}_i, \mathfrak{q}_i)}$$

$$\boxed{\Delta_i|_{crit} \sim X^i(r_h)}$$

[Benini-Hristov-AZ]

Part III

Interpretation and Conclusions

A. Statistical ensemble

Δ_a can be seen as chemical potential in a macro-canonical ensemble defined by the supersymmetric index

$$Z = \mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_a J_a} e^{-\beta H}$$

so that the extremization can be rephrased as the statement that the black hole is electrically charged

$$\frac{\partial}{\partial \Delta} log Z \sim i < J >= i\mathfrak{q}$$

- Similarities with Sen's entropy formalism based on AdS_2.

B. Attractor mechanism

The BPS equations at the horizon imply that the gauge supergravity quantity

$$\mathcal{R} = \left(F_\Lambda \mathfrak{n}^\Lambda - X^\Lambda \mathfrak{q}_\Lambda \right) , \qquad F_\Lambda = \frac{\partial \mathcal{F}}{\partial X^\Lambda}$$

with $(\mathfrak{q}, \mathfrak{n})$ electric and magnetic charges, is extremized with respect to the scalar fields at the horizon and its critical value gives the entropy

Under $X^\Lambda \to \Delta^\Lambda$

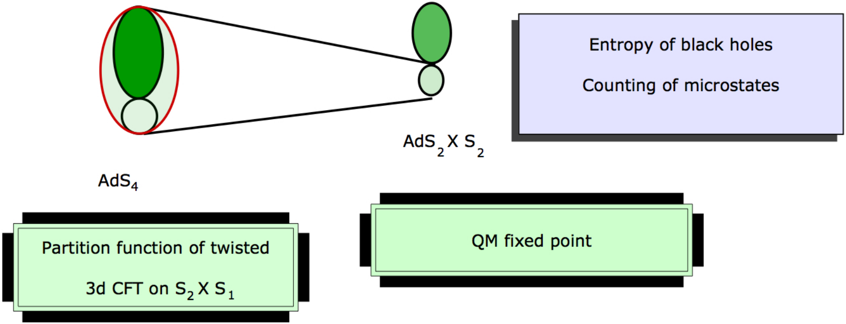
$$\mathcal{F} = 2i\sqrt{X^0 X^1 X^2 X^3} \equiv \mathcal{V}_{BA}(\Delta)$$

$$i\mathcal{R} = \sum -\frac{\mathfrak{n}_\Lambda}{X^\Lambda}\sqrt{X^0 X^1 X^2 X^3} - iX^\Lambda \mathfrak{q}_\Lambda \equiv \mathcal{I}(\Delta)$$

[Benini-Hristov-AZ]

C. The IR superconformal QM

Recall the cartoon



The IR superconformal QM

RG flow with symmetry enhancement at the horizon AdS_2

$$\mathrm{QM}_1 \to \mathrm{CFT}_1$$

Take a purely magnetically charged black hole. Running scalars $X_i \to \Delta_i$ reflect the mixing of R-symmetry with flavor symmetries

$$\mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_i J_i} \equiv \mathrm{Tr}_{\mathcal{H}}(-1)^R$$

where $R = F + \Delta_i J_i$ is a trial R-symmetry of the system.

R-symmetry mixing

The mixing reflects exactly what's going on in the bulk. The graviphoton field strength depends on r

$$T_{\mu\nu} = e^{K/2} X^{\Lambda} F_{\Lambda,\mu\nu}$$

suggesting that the R-symmetry is different in the IR and indeed

$$\boxed{\Delta_i|_{crit} \sim X^i(r_h)}$$

R-symmetry extremization

Some QFT extremization is at work? symmetry enhancement at the horizon AdS_2

$$\mathrm{QM}_1 \to \mathrm{CFT}_1$$

and

$$\mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_i J_i}\big|_{crit} \equiv \mathrm{Tr}_{\mathcal{H}}(-1)^{R_{exact}} \equiv \mathrm{Tr}_{\mathcal{H}} 1$$

$R_{exact} = F + \Delta_i J_i|_{crit}$

- ground states are singlets of the superconformal group ($R = 0$)
- index is extremized at the exact R-symmetry at the superconformal point and is the number of states ($R = 0$)
- Natural thing to extremize: in even dimensions central charges are extremized, in odd partition functions...

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS_4 black holes to a microscopic counting of states.

- first time for AdS black holes in four dimensions

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- first time for AdS black holes in four dimensions

But don't forget that we also gave a general formula for the topologically twisted path integral of 2d (2,2), 3d $\mathcal{N} = 2$ and 4d $\mathcal{N} = 1$ theories.

- Efficient quantum field theory tools for testing dualities.
- With many field theory questions/generalizations

Thank you for the attention !