# Phases of $\mathcal{N} = 1$ Adjoint SQCD in $2 + 1$ Dimensions

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# Overview

# $\mathcal{N} = 1$ Supersymmetry in $d = 2 + 1$

- Supercharges form a Majorana spinor $Q_\alpha$.
- Two main characters:
  - Vector multiplet $V_\alpha = i(\not{A}\theta)_\alpha + \frac{1}{2}\bar{\theta}\theta\lambda_\alpha$.
  - Real multiplet $\Phi = X + \bar{\theta}\psi + \frac{1}{2}\bar{\theta}\theta F$.
- Supersymmetry is "real", so there are no non-renornalization theorems. Superpotential can receive perturbative corrections,
- Vacuum energy is still bounded by zero from below.
- Witten index can be introduced.
- There might be experimental realizations of $d = 2 + 1$ minimal supersymmetry (Li, Vaezi, Mendl, Yao '17).

# $\mathcal{N} = 1 \; SU(N)_k$ Vector Multiplet

$$\mathcal{L} = -\frac{1}{4g^2} TrF^2 + iTr\lambda \not{D}\lambda + \frac{k}{4\pi} Tr\left(AdA - \frac{2i}{3}A^3\right) - \frac{kg^2}{2\pi} Tr\lambda\lambda$$

- Witten index (Witten '99):

$$I_W = \frac{1}{(N-1)!} \prod_{j=-N/2+1}^{N/2-1} (k-j) = \begin{cases} \neq 0, & \text{if } k \geq N/2. \\ = 0, & \text{if } 0 \leq k < N/2. \end{cases}$$

- $k \geq N/2$: SUSY preserved, $SU(N)_{k-\frac{N}{2}}$ in the IR.
- $0 \leq k < N/2$: SUSY is broken, $G_\alpha \; + \; U(\frac{N}{2}-k)_{\frac{N}{2}+k,N}$ in the IR (Gomis, Komargodski, Seiberg '17).

# Vector Multiplet with an Adjoint Matter Multiplet

Vector multiplet $(A, \lambda)$, matter multiplet $(X, \psi)$.

$$\mathcal{L} = \frac{1}{4g^2} TrF^2 + iTr\lambda \not{D} \lambda + iTr\psi \not{D} \psi + Tr(DX)^2 +$$
$$+\frac{k}{4\pi} Tr \left( AdA - \frac{2i}{3} A^3 \right) - \frac{kg^3}{2\pi} Tr\lambda\lambda + \sqrt{2} ig Tr[\lambda, X]\psi.$$

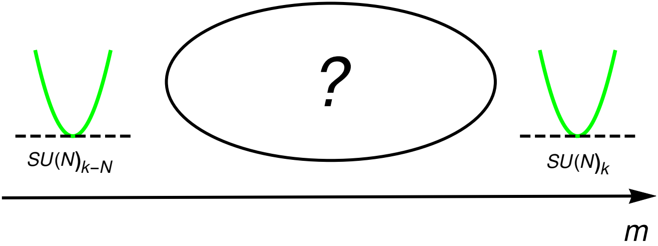
- $\mathcal{N} = 1$ deformation $Tr(m^2 X^2 + m\psi\psi)$.
- For $m = -\frac{kg^2}{2\pi}$ SUSY is enhanced to $\mathcal{N} = 2$.

# Large Mass Asymptotic Phase: $k \geq N$

- Matter multiplet can be integrated out

$$SU(N)_k \to \begin{cases} SU(N)_{k+N/2}, & m \to +\infty \\ SU(N)_{k-N/2}, & m \to -\infty \end{cases}$$

- $k \pm N/2 \geq N/2$ - in both limits $m \to \pm\infty$ physics of the "large $k$" phase.
- Witten index jumps at the point $m = 0$.

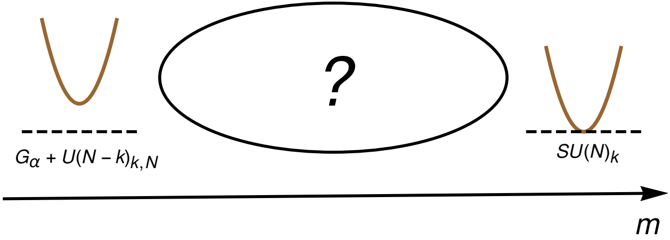


# Large Mass Asymptotic Phase: $0 < k < N$

- Matter multiplet can be integrated out

$$SU(N)_k \to \begin{cases} SU(N)_{k+N/2}, & m \to +\infty \\ SU(N)_{k-N/2}, & m \to -\infty \end{cases}$$

- $|k - N/2| < N/2$ - in the limit $m \to -\infty$ SUSY is broken.
- Witten index jumps at the point $m = 0$.



# Classical Moduli Space of Vacua at $m = 0$

$$X = \begin{pmatrix} X_1 & 0 & 0 & \dots & 0 \\ 0 & X_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & X_{N-1} & 0 \\ 0 & 0 & 0 & \dots & X_N \end{pmatrix}, \quad \sum_{i=1}^{N} X_i = 0.$$

Moduli space: $\mathbb{R}^{N-1}/S_N,$ where $S_N$ is Weyl group of $SU(N)$.

# Classical Abelian Vacua

- Abelian vacua: $X_i \neq X_j$ for $i \neq j$.
- Unbroken gauge group: $U(1)^{N-1}$.
- IR TQFT:

$$\frac{K_{ij}}{4\pi}\int A^i dA^j,\ K = k\begin{pmatrix} 2 & -1 & 0 & \ldots & 0 \\ -1 & 2 & -1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & -1 & 2 \end{pmatrix}.$$

- Defined up to an $SL(N-1,\mathbb{Z})$ transformation

$$K \to MkM^T,\ \vec{A} \to M\vec{A},\ M \in SL(N-1,\mathbb{Z}).$$

- For $k = 1$ this TQFT is dual to $U(1)_{-N}$ Chern-Simons theory:

$$\frac{K_{ij}}{4\pi}\int A^i dA^j \leftrightarrow -\frac{N}{4\pi}\tilde{A}\wedge d\tilde{A}.$$

# Classical Non-Abelian Vacua

- Non-Abelian vacua: $X_i = X_j$ for some $i \neq j \to L$ blocks of the size $S_l \times S_l$, $l = 1, ..., L$.

$$X = \begin{pmatrix} S_1 \times S_1 & & & \\ & S_2 \times S_2 & & \\ & & ... & \\ & & & S_L \times S_L \end{pmatrix}.$$

- Unbroken gauge group: $S[U(S_1) \times ... \times U(S_1)]$.
- IR TQFT if the gauge group was $U(N)$:

$$U(S_1)_{k,k} \times U(S_2)_{k,k} \times ... \times U(S_L)_{k,k}.$$

- Use Lagrange multiplier to come back to the $SU(N)$:

$$\frac{1}{2\pi} \wedge \sum_{l=1}^{L} S_l TrA_l.$$

# Semiclassical Moduli Space of Vacua

- Non-trivial superpotential $\mathcal{W}(X_i)$ can be perturbatively generated and lift the moduli space.
- To detect the appearance of new vacua, it is convenient to use the large $X$ expansion, assuming first that $X_{ij} \neq 0$ (violated at the singular loci).
- Assuming the uniform scaling $X_{ij} \sim X$ and choosing the suitable gauge, we get by dimensional analysis for the scalar potential

$$V^{(L)}(X) = g^{2L-2} \sum_{n>0} d_{n;L} \frac{(kg^2)^{2n}}{(gX)^{L+2n-4}}, \quad V = \sum_L V^{(L)}(X).$$

# Semiclassical Moduli Space of Vacua: one-loop order

- One-loop contribution to the scalar potential vanishes

$$V^{(1)} = 0.$$

- Upon the integration out of the massive charged fermions, CS level matrix $K$ is not renormalized.

# Semiclassical Moduli Space of Vacua: two-loop order

- The scalar potential given above corresponds to the superpotential

$$\mathcal{W}(X) = kg^3 \sum_{L>1} g^L \sum_{n>0} c_{n;L} \frac{(kg^2)^{2n-2}}{g^{2n} X^{L+2n-5}}.$$

- Any given term in the $1/X$ expansion receives contributions from finitely many loop orders in perturbation theory.
- The leading term is at $L = 2$, $n = 1$, and scales linearly:

$$\mathcal{W} \sim X.$$

- The two-loop superpotential has been computed (Armoni, Hollowood '05, '06)

$$\mathcal{W} = -\sum_{ij} g^3 k \sqrt{g^2 k^2 + X_{ij}^2}.$$

# Semiclassical Moduli Space of Vacua: two-loop order

- In the "far zone" $X \gg gk$, the only reliable information is the linear term:

$$\mathcal{W} = -g^3 k \sum_{ij} |X_{ij}|.$$

- We see that the classical moduli space is lifted at two loops.

# Semiclassical Abelian Vacua near m=0

- Consider now the small mass deformation of the theory

$$\mathcal{W} = -\sum_{ij} g^3 k \sqrt{g^2k^2 + X_{ij}^2} + m\sum_i X_i^2 + \lambda \sum_i X_i .$$

- F-term equations in the "far zone" $X \gg gk$ take the form

$$-g^3 k \sum_j sgn(X_{ij}) + mX_i + \frac{1}{2}\lambda = 0, \sum_i X_i = 0.$$

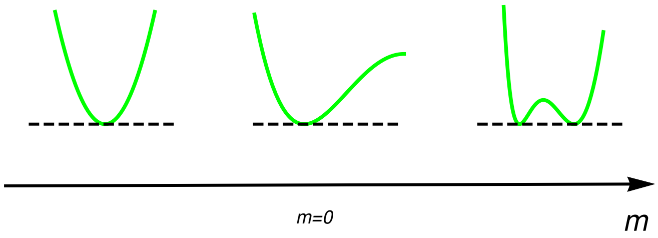
- For small negative $m$ there is no solution.
- For small positive $m$ a solution exists given by (up to an action of the Weyl group)

$$X_i = \frac{g^3 k}{m}(N+1-2i).$$

This is a supersymmetric vacuum with $U(1)^{N-1}$ Chern-Simons theory.

# Semiclassical Abelian Vacua near m=0

Intuitive picture: for $m < 0$ there is one vacuum,potential increases in all directions, for $m = 0$ a flat direction opens up, and for $m > 0$ a new vacuum comes from the infinity.



# Semiclassical (Non)-Abelian Vacua at large $k$.

- Large $k$ is another weakly coupled regime. The critical points of the superpotential are given by the solutions of

$$mX_i + \frac{1}{2}\lambda = g^3 k \sum_i \frac{X_{ij}}{\sqrt{g^2k^2 + X_{ij}^2}}, \sum_i X_i = 0.$$

- There is clearly the solution $X_i = 0$.
- Solutions with $X_i \neq 0$ for some $i$ exist only for $\frac{m}{g^2} \in (0, N)$. Assuming $m$ to be small, they are given by

$$X_I = \frac{g^3 k}{m}[(S_{I+1} + ... + S_L) - (S_1 + ... + S_{I-1})],$$

where $S_I$ is the size of the $I$th block. There are $2^{N-1}$ vacua , corresponding to the ordered partitions (or compositions) of $N$.

# Semiclassical (Non)-Abelian Vacua at large $k$.

- For convenience we switch from $SU(N)$ to $U(N)$.
- In order to understand the IR theory in each vacuum, we need the fermion mass matrix. It is block-diagonal with $S_I \times S_I$ blocks, $I = 1, ..., L$. Eigenvalues of the $I$th block are

$$(-g^2 S_I + m, -g^2 S_I + m, ..., -g^2 S_I + m, m),$$

  such that all the fields apart from the decoupled mode have negative masses.
- The resulting infrared TQFT is then given by

$$U(S_1)_{k-S_1,k} \times U(S_2)_{k-S_2,k} \times ... \times U(S_L)_{k-S_L,k}.$$

# Matching the Witten Index

- The resulting infrared TQFT:
  $U(S_1)_{k-S_1,k} \times U(S_2)_{k-S_2,k} \times \ldots \times U(S_L)_{k-S_L,k}$
- It then follows that every vacuum carry the index

$$\prod_I \frac{k!}{S_I!(k-S_I)!}.$$

- When the total index is being computed, the contributions of different vacua must be weighted by a sign (Witten, '82)

$$(-1)^{\sum_I (S_I-1)} = (-1)^{N-L}.$$

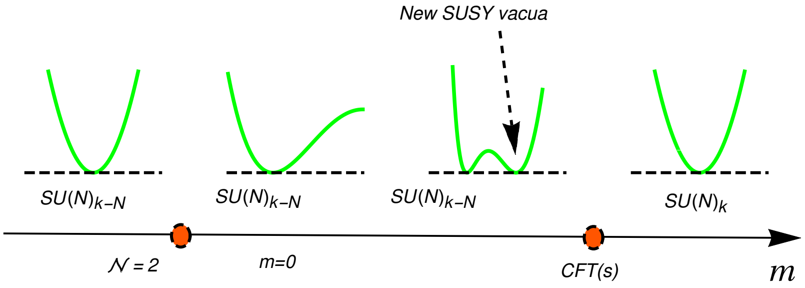
- The total index is then given by

$$I = \sum_P (-1)^{N-L} \prod_I \frac{k!}{S_I!(k-S_I)!}.$$

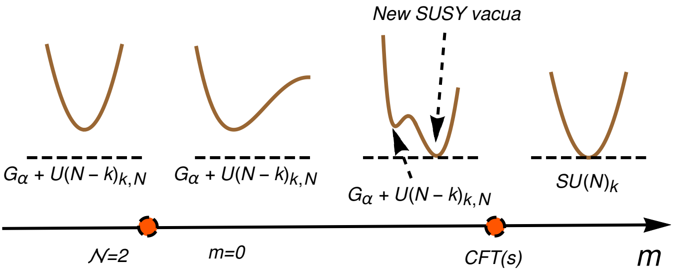
- One can proof the following combinatorial identity

$$\sum_P (-1)^{N-L} \prod_I \frac{k!}{S_I!(k-S_I)!} = \frac{(N+k-1)!}{N!(k-1)!} = I_{SU(N)_k}.$$

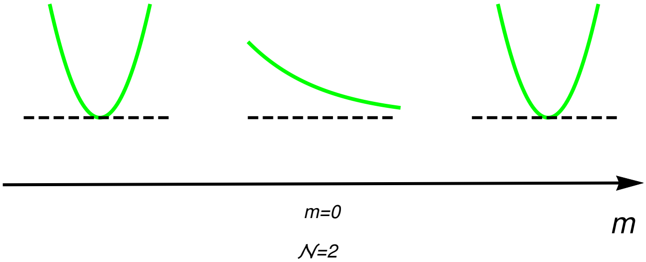
# Phase diagram for $k \geq N$



# Phase diagram for $0 < k < N$



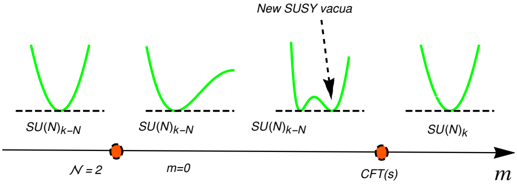
# Phase diagram for $k = 0$

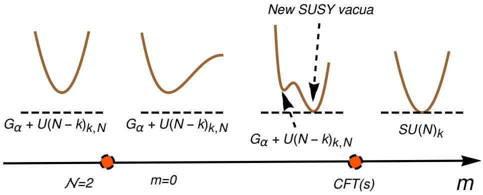


# Further Developments

- The problem of $\text{SQCD}_3$ with fundamentals was considered in the same work. A duality between $SU$ and $U$ theories is suggested.
- A similar class of models, but with the tree-level superpotential was considered in Benini, Benvenuti '18. They proposed another duality, involving the gauge singlet.
- Gaiotto, Komargodski, Wu '18 studied $\mathcal{N} = 1$ theories with time reversal symmetry, which sometimes provides the existence of exact moduli spaces.
- Benini, Benvenuti '18 observed the global symmetry enhancement in $\mathcal{N} = 1$ QED.

# Instead of Conclusion



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# Thank you for your attention!