3d dualities and Weyl group symmetry

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Supersymmetric theories,
dualities and deformations
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Introduction

A good strategy to derive new dualities is to take limits and deformations of existing ones

It is especially interesting to find relations between dualities in different dimensions

Rich interplay between duality and symmetries

Outline

Review of USp(2N) dualities and E7 surprise

Circle reduction of 4d dualities

Monopole superpotentials and zero modes

Real mass and Higgs flows

New webs of USp(2N)/U(N) dualities

USp(2N) theories in 4d

The theory we consider has:

- $USp(2N)$ gauge group
- 8 fundamental flavors Q
- 1 (totally) antisymmetric field A
- superpotential $W = 0$

The global symmetry is $SU(8) \times U(1) \times U(1)_R$

The theory has a large number of duals [Spiridonov,Vartanov]
The rank 1 case was studied by [Gaiotto,Dimofte]
Higher rank generalizations are due to [Razamat,Zafrir]

USp(2N) dualities in 4d

There are 4 sets of dual phases:

	$USp(2N)$	$SU(8)$	$U(1)_R$
Q	$2N$	8	$1/2$
A	$N(2N-1)-1$	1	0

$$W_A = 0$$

	$USp(2N)$	$SU(8)$	$U(1)_R$
q	$2N$	$\bar{8}$	$1/2$
a	$N(2N-1)-1$	1	0
$M^{(j)}$	1	28	1

$$W_B = M^{(j)} qqa^j$$

[Intriligator,Pouliot]

	$USp(2N)$	$SU(4)$	$SU(4)$	$U(1)_b$	$U(1)_R$
q	$2N$	$\bar{4}$	1	1	$1/2$
p	$2N$	1	4	-1	$1/2$
a	$N(2N-1)-1$	1	1	0	0
$M^{(j)}$	1	4	$\bar{4}$	0	1

$$W_C = M^{(j)} qpa^j$$

[Seiberg]

	$USp(2N)$	$SU(4)$	$SU(4)$	$U(1)_b$	$U(1)_R$
q	$2N$	4	1	1	$1/2$
p	$2N$	1	4	-1	$1/2$
a	$N(2N-1)-1$	1	1	0	0
$M^{(j)}$	1	1	6	-2	1
$N^{(j)}$	1	6	1	2	1

$$W_D = M^{(j)} qqa^j + N^{(j)} ppa^j$$

[Csaki,Schmaltz,Skiba,Terning]

E7$\times$U(1) surprise

[Razamat,Zafrir]

In total there are **1+1+35+35=72** dual phases

For even rank they can be deformed so that they become self dual

self duality $\leftrightarrow$ discrete global symmetry

Global symmetry enhances from $SU(8) \times U(1)$ to $E_7 \times U(1)$

The enhancement can be checked by expanding the superconformal index and rearranging the gauge invariant operators into irreps. of E_7

Weyl group symmetry

The Weyl group of E_7 has an action on the fugacities with stabilizer the group of permutations of the 8 flavors

$$\frac{|W(E_7)|}{|W(A_7)|} = 72$$

The dualities are implemented by reflections in the roots of E_7 which are not in $SU(8)$

$$\mathbf{133} = \mathbf{63} \oplus \mathbf{70}$$

Reduction of duality to 3d

We can put the theories on $\mathbb{R}^3 \times S^1$ and take the limit $r \to 0$ but the naive dimensional reduction does not give rise to a 3d duality!

To correctly reduce the 4d duality one has to modify the limit procedure in the following ways:
[Aharony,Razamat,Seiberg,Willett]

- the scalar fields coming from the holonomy of the gauge field around the circle are periodic
  $\Rightarrow$ compact Coulomb branch
- 4d instantons *can* generate a non-perturbative superpotential on the Coulomb branch of the effective 3d theories [Seiberg,Witten]

$$W = \eta Y$$

Monopole superpotentials

- This superpotential arises due to the presence of 2 zero modes of the Dirac operator in a 4d instanton background (KK monopole)
- It can be seen as a contribution coming from a fundamental monopole associated to the affine root of the algebra
- Global symmetries can be anomalous in 4d and dualities hold only when anomaly cancellation is satisfied
- Requiring that the monopole superpotential has the correct charges imposes the same constraints as the 4d anomaly cancellation

Counting of zero modes

The number of zero modes associated to every matter field can be computed using the Callias index of the Dirac operator on $\mathbb{R}^3 \times S^1$ in a KK monopole background:

- the adjoint carries 2 zero modes for each unit of magnetic flux
- the fundamental and antisymmetric have no zero modes

The KK monopole superpotential is indeed generated and the duality is preserved!

Remark: the theory is only *effectively* 3-dimensional

3d partition functions

The duality can be checked at the level of the 3d partition function on S_b^3 by reducing the 4d index [Rains]

The partition function is a *hyperbolic hypergeometric* integral which depends on the complex variables:

- $\mu \in \mathbb{C}^8$, mass parameters of the fundamentals
- $\tau \in \mathbb{C}$, mass parameter of the antisymmetric

subject to the **balancing condition**:

$$2(N-1)\tau + \sum_{r=1}^{8} \mu_r = 4\omega \qquad \omega = \frac{i}{2}(b + 1/b)$$

$\{Re(\mu), Re(\tau)\}$ = real masses, $\{Im(\mu), Im(\tau)\}$ = R-charges

Integral identities

Several mathematical identities are known for this type of hypergeometric integrals in the literature: [van de Bult]

$$Z_{USp}(\mu;\tau) = \prod_{j=0}^{N-1} \prod_{1\leq r<s\leq 4} \Gamma_h(j\tau+\mu_r+\mu_s) \times \prod_{5\leq r<s\leq 8} \Gamma_h(j\tau+\mu_r+\mu_s)\, Z_{USp}(\tilde{\mu};\tau)$$

$$\tilde{\mu} = \left\{ \begin{array}{ll} \mu_r+\zeta & r=1,..,4 \\ \mu_r-\zeta & r=5,..,8 \end{array} \right\} \qquad 2\zeta = \sum_{r=5}^{8} \mu_r - 2\omega + (N-1)\tau$$

By combining this master equation with permutations of the mass parameters we obtain identities between all the dual phases.

Real mass flow

The previous identity is compatible with the assignment of masses as:

$$\mu_7 \to \mu_7 + M \qquad \mu_8 \to \mu_8 - M$$

After taking the $M \to \infty$ limit we can use the balancing condition to remove the dependence on $\mu_{7,8}$ so that the remaining parameters are unconstrained (vanishing monopole superpotential)

$$Z_{USp}(\mu;\tau) = \prod_{j=0}^{N-1} \prod_{1 \leq r < s \leq 4} \Gamma_h(j\tau + \mu_r + \mu_s)\Gamma_h(j\tau + \mu_5 + \mu_6) \times \Gamma_h\left(4\omega - (2N-2+j)\tau - \sum_{r=1}^{6} \mu_r\right) Z_{USp}(\tilde{\mu};\tau)$$

The hypergeometric integrals have $W(D_6)$ symmetry.

Breaking of Weyl group symmetry

The choice of real masses selects the direction
$v = (0,0,0,0,0,0,1,-1)$ in the root space of E_7

Reflections in the roots orthogonal to v generate a parabolic subgroup of the Weyl group of E_7 which acts as the unbroken symmetry group of the partition function.

Accordingly, the number of dual phases is:

$$\frac{|W(D_6)|}{|W(A_5)|} = 32$$

Higgs flow and new USp(2N)/U(N) dualities

We can also combine a mass flow in the direction $(1, 1, 1, 1, -1, -1, -1, -1)$ with an Higgs flow

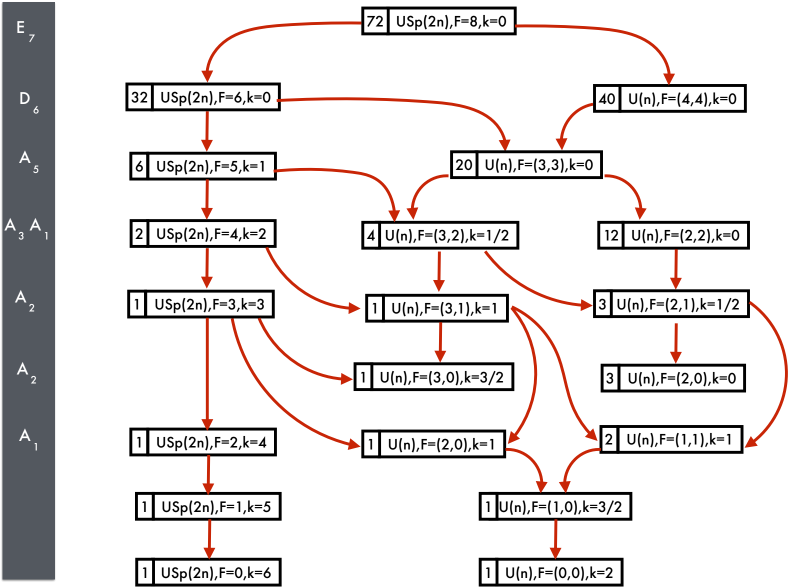
$$\sigma_i \to \sigma_i + M$$

The shift breaks $USp(2N)$ with 8 fund. and 1 antisymm. into $U(N)$ with 4 flavors and 1 adj. + superpotential $W = Y + \tilde{Y}$

Remark: Naively this superpotential for $U(N)$ + adjoint should not be generated, but the UV completion of the effective theory is $USp(2N)$ with antisymm.

- partition function has $W(D_6)$ symmetry with $\frac{|W(D_6)|}{|W(A_3)^2|} = 40$ dual phases.
- The two types of flow can be mapped by $W(E_7)$:
  $\Rightarrow$ duality between the $USp(2N)$ and $U(N)$ theories.

General scheme



Conclusions

- we found the 3d reduction of dualities of $USp(2N)$ theories with 8 flavors and 1 antisymmetric
- by mass + Higgs flows we obtain several new $USp(2N)/U(N)$ dualities
- many of these theories exhibit global symmetry enhancement
- the action of the E_7 Weyl group is manifest on the real masses and governs both the reduction of the dualities by flows and also the symmetry enhancements

Outlook

- brane realization
- other flows leading to different parabolic subgroups
- theories with power law superpotential for A
- confining theories with 6 fundamentals
- Hilbert series and superconformal/twisted index