Three dimensional SQCD and mirror symmetry

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ICTP

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S.G. Work in progress

Infrared dualities

Infrared dualities: two different field theories have the same long distance dynamics. They are ubiquitous in supersymmetric theories (several examples without supersymmetry in 3d) and are often essential in providing insights about nonperturbative effects.

In 4d $SU(N)$ SQCD with $N_f < \frac{3N}{2} = SU(N_f - N)$ with N_f flavors.

At present there is no algorithm which allows to systematically extract dualities: starting from a set of well-established examples (e.g. Seiberg duality in 4d or mirror symmetry in 3d), we would like to be able to change the matter content and interactions at will, keeping track of how the dual theory evolves in the process.

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$\mathcal{N} = 2$ mirror symmetry for abelian theories

Seen as a $\mathcal{N} = 2$ theory, any $\mathcal{N} = 4$ gauge theory has a chiral multiplet in the adjoint representation coupled to all other matter fields through a cubic superpotential term. Giving mass to it generates a (relevant) quartic interaction.

Starting from the $\mathcal{N} = 4$ abelian duality:

$\mathcal{W} = \widetilde{Q}_i \Phi Q^i$ $\qquad\qquad$ $\mathcal{W} = \sum_i S_i \tilde{q}_i q_i - \Psi(S_1 + \cdots + S_N)$

We can obtain $\mathcal{N} = 2$ SQED adding a singlet λ and the superpotential $\delta\mathcal{W} = \lambda\Phi$. On the dual side this is mapped to $\delta\mathcal{W} = \lambda\Psi$.

Aharony, Hanany, Intriligator, Seiberg, Strassler '97.

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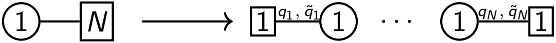

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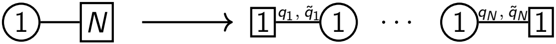

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Monopole Duality

We use the following duality: Benini, Benvenuti, Pasquetti '17.

1. $U(N_c)$ SQCD with N_f flavors and $\mathcal{W} = \mathfrak{M}^+$
2. $U(N_f - N_c - 1)$ SQCD with N_f flavors and $\mathcal{W} = M_j^i \widetilde{Q}_i Q^j + \mathfrak{M}^- + X\mathfrak{M}^+$

For $N_f = N_c + 1$ theory 2 becomes a WZ model with superpotential

$$\mathcal{W} = X\text{Det}(M)$$

Chiral operators are mapped as follows:

$$\widetilde{Q}_i Q^j \leftrightarrow M_i^j; \quad \mathfrak{M}^- \leftrightarrow X$$

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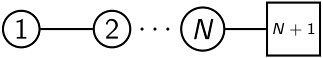
Sequential confinement

Consider the $\mathcal{N} = 4$ theory (usually called $T(SU(N+1))$)



and turn on superpotential terms $\mathcal{W} = \mathfrak{M}_i^+$ for all the gauge groups.

Using monopole duality, we find that all the groups confine and we get a field M in the adjoint of $SU(N+1)$ plus singlets X_i.

$$\mathcal{W} = \sum_{i=2}^{N+1} X_i \,\mathrm{Tr}(M^i) + \sum_i \alpha_i X_i$$

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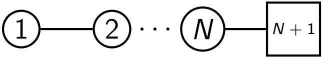

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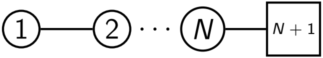

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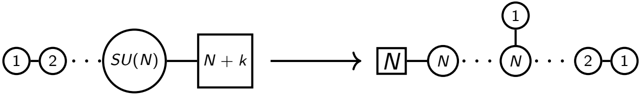
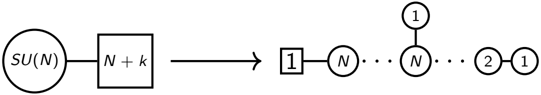
Mirror dual of $\mathcal{N} = 2$ SQCD

Start from the known $\mathcal{N} = 4$ mirror pair



If we now turn on the monopole deformation at nodes $U(1) \ldots U(N-1)$ on the l.h.s. all the gauge groups except $SU(N)$ confine and we get $\mathcal{N} = 2$ SQCD! On the dual side the deformation is mapped to an off-diagonal mass matrix.



Mapping of chiral operators

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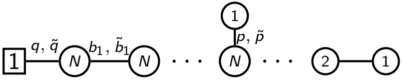

$$\mathcal{W} = \tilde{q}\phi^N q + \sum_i \alpha_i \tilde{q}\phi^{N-i} q + \dots$$

- Mesons and baryons of SQCD are mapped to monopoles.
- α_i map to Casimirs of the meson matrix ($\sim \mathrm{Tr}\, M^k$).
- The monopole of SQCD is mapped to $q b_1 \dots p\tilde{p} \dots \tilde{b}_1 \tilde{q}$.

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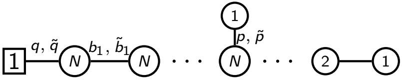

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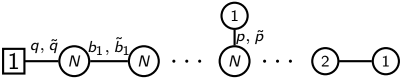

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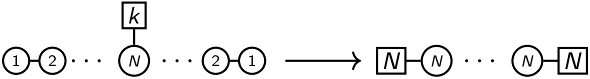

Turning on the monopole deformation at both tails we find a dual description of adjoint SQCD with zero superpotential

Problem: in the dual theory $R(\Phi) = R(\widetilde{Q}Q)$, meaning there is an hydden symmetry which mixes with R-symmetry!

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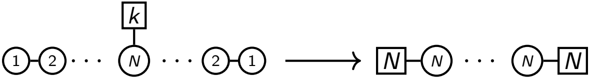

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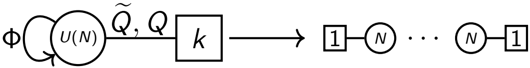

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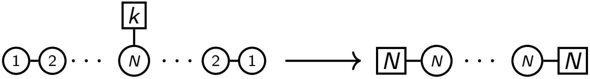

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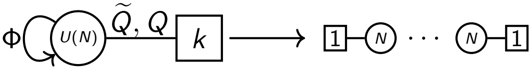

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Emergent symmetries and $U(2)$ SQCD

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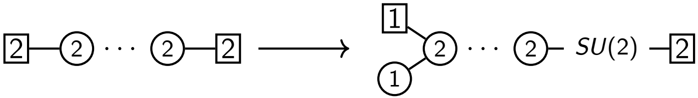

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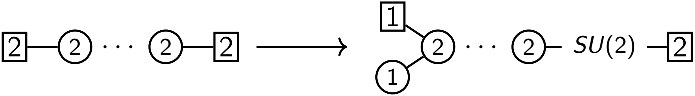

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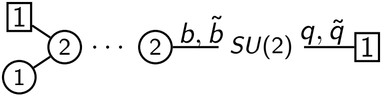

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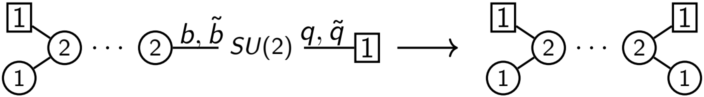

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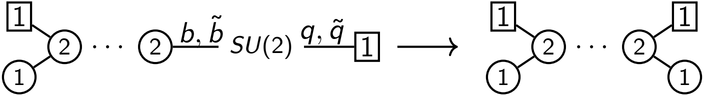

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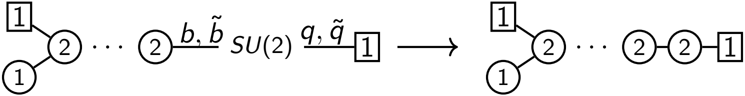

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Theory with one flavor and duality appetizer

The mirror we find of $SU(2)$ adjoint SQCD with one flavor is a $U(1)$ theory with two flavors and superpotential

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Our construction can be generalized to introduce an arbitrary number of adjoint matter fields (subtleties due to emergent symmetries).

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