Flowing From 16 to 32 Supercharges

Supersymmetric Theories, Dualities and Deformations

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Based on:
M.B., Z. Laczko, and T. Nishinaka [arXiv:1807.02785[hep-th]]

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Outline

- Motivation and review
- UV starting points
- The simplest RG flow
- Generalizations
- Conclusions and open questions

Motivation and Review

Goals:

(i) Find a more unorthodox way to construct theories with 4D $\mathcal{N} = 4$ SUSY.

(ii) Find a more controlled set of examples in which to study SUSY enhancement.

Why:

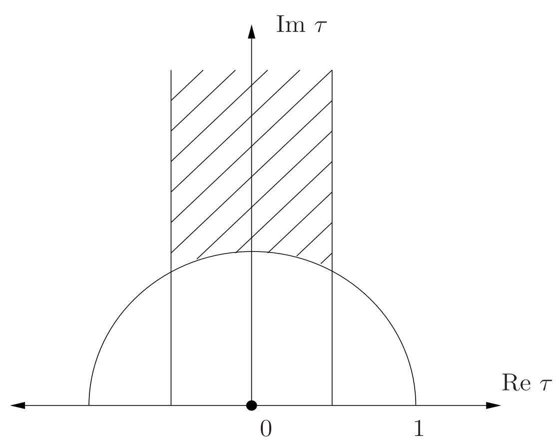
(i) Is the known space of $\mathcal{N} = 4$ theories complete?

(ii) Given UV data and deformations, can we predict IR SUSY enhancement?

Motivation and Review (cont...)

• We often assume the list of $\mathcal{N} = 4$ theories is known: take some lie algebra, $\mathfrak{g}$, an $\mathcal{N} = 1$ vector multiplet, three adjoint chiral multiplets, and gauge.

• The gauge coupling, τ, is exactly marginal.



Motivation and Review (cont...)

• Various subtleties to do with the global structure of the gauge group and extended operators [Aharony, Seiberg, Tachikawa], [Argyres, Martone], [García-Etxebarria].

• But, these discussions do not affect the *local* operator content. Are there $\mathcal{N} = 4$ theories that are more exotic?

• If $\exists$ such (local) $\mathcal{T}$, then

(i) $\mathcal{T}$ has no weak coupling limit.

(ii) $\mathcal{T}$ has an exactly marginal deformation [Dolan, Osborn]

(iii) If $\mathcal{T}$ has an odd dimensional vacuum moduli space $\Rightarrow$ global Witten anomaly for $su(2) \subset su(4)_R$.

Motivation and Review (cont...)

- How would we build such theories (*if* they exist!) in QFT?

- Starting in CFT via the bootstrap

$$\mathcal{C} = \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_N(x_N)\rangle \ , \tag{1}$$

perhaps using chiral algebras à la **[Beem et. al.]** and defects **[Liendo, Meneghelli]**. Such approaches have been successful in the context of 2D CFT.

- We can also try to construct such theories via the RG flow (without ever explicitly discussing free fields)

- This necessarily implies $\mathcal{N} = 4$ SUSY is emergent/accidental.

Motivation and Review (cont...)

• Basic idea behind accidental symmetry. Deform UV as in (2) follow to IR and find

$$S_{IR} = S_{CFT} + \int d^4x \tilde{\lambda}^i \tilde{\mathcal{O}}_i \ , \tag{2}$$

with $\tilde{\mathcal{O}}_i$ irrelevant (so that $\tilde{\lambda}_i \to 0$ as we flow to IR) and some of these operators break IR symmetries.

3D: $\mathcal{N} = 0 \to \mathcal{N} = 1, 2$ [Balents, Fisher, Nayak], [Grover, Sheng, Vishwanath], [Lee], [Thomas], $\cdots$. $\mathcal{N} = 3 \to \mathcal{N} = 6, 8$ [ABJM],$\cdots$. $\mathcal{N} = 1 \to \mathcal{N} = 2$ [Gaiotto, Komargodski, Wu], [Benini, Benvenuti]

4D: $\mathcal{N} = 1 \to \mathcal{N} = 2$ [Maruyoshi, Song], [Benvenuti, Giacomelli], [Aghaei, Amariti, Sekiguchi],$\cdots$, $\mathcal{N} = 2 \to \mathcal{N} = 4$ [Argyres, Lotito, Lu, Martone]

Motivation and Review (cont...)

• If via a vev, then UV could start from $\mathcal{N} = 3, 2, 1, 0$.

• If via a relevant deformation

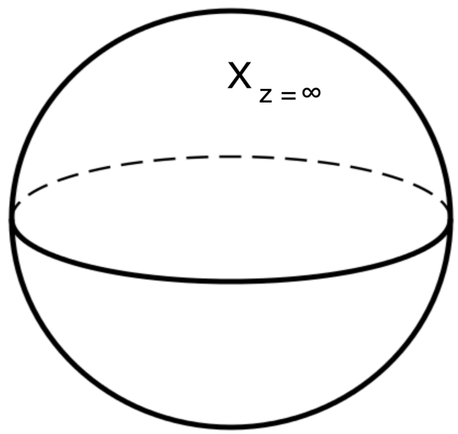
$$\delta S \sim \int d^4 x \lambda \mathcal{O} \ , \tag{3}$$

then $\mathcal{N} = 2, 1, 0$.

• Here we will choose the latter option and $\mathcal{N} = 2$. The reason is these theories are fairly controlled and *still* very exotic.

UV Starting Points

• UV starting points are 4D $\mathcal{N} = 2$ SCFTs arising via twisted compactifications of A_{N-1} (2,0) theory on $\mathcal{C} = \mathbb{CP}^1$ with one "irregular" puncture at $z = \infty$.



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• Progress in finding space of irregular punctures [Xie], [Bonelli, Maruyoshi, Tanzini], [M.B., Giacomelli, Papageorgakis, Nishinaka], [Xie, Ye], [Xie, Wang]. Generalizes [Gaiotto]. Wild frontier!

UV Starting Points (cont...)

• Useful to first compactify $(2,0)$ theory on $S^1 \to$ 5D maximal SYM. Twisted vector multiplet furnishes

$$\Phi_z = z^{\ell-2} T_{\ell-2} + z^{\ell-3} T_{\ell-3} + \cdots + T_0 + \frac{1}{z} T_{-1} + \cdots \ , \qquad (4)$$

where $T \in \mathcal{M}_{N \times N}$ traceless (diagonal) and $\ell > 1$ in $\mathbb{Z}$.

• Part of solution to Hitchin's equations: gives rise to Higgs branch of 3D mirror of S^1 reduction of 4D theory $\Rightarrow$ Coulomb branch of direct reduction and 4D theory too.

• For example, SW curve from

$$\det(x - \Phi_z) = 0 \ . \qquad (5)$$

UV Starting Points (cont...)

- Turns our theories can be specified by Young diagrams

$$
\begin{aligned}
T_i \leftrightarrow Y_i &= [n_{i,1}, n_{i,2}, \cdots, n_{i,k_i}] \ , \quad n_{i,a} \geq n_{i,a+1} \in \mathbb{Z}_0 \ , \\
\sum_{a=1}^{k_i} n_{i,a} &= N \ . \qquad (6)
\end{aligned}
$$

where having some columns of height > 1 implies a degeneracy—so-called "Type III" theories [Xie].

- Our theories of interest will have

$$
Y_{1,0} = [n, \cdots, n] \ , \quad Y_{-1} = [n, \cdots, n, n-1, 1] \ , \qquad (7)
$$

where $n \geq 2$, $k_1 = k_0 \geq 3$ and $k_{-1} = k_1 + 1 \geq 4$ and so $N = nk$.

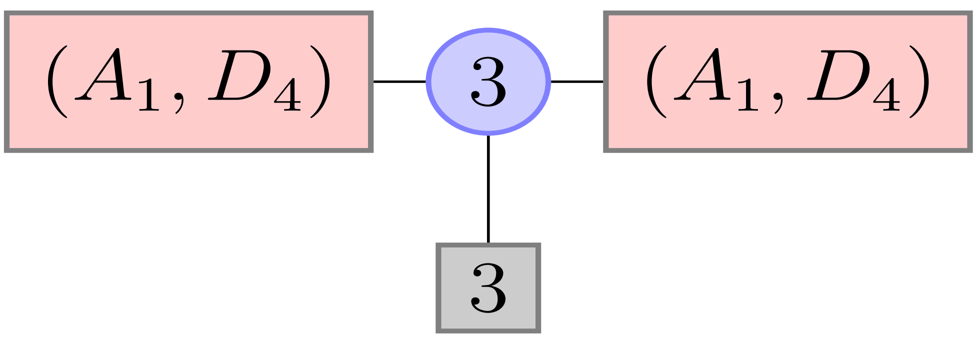
The Simplest RG Flow

- The simplest RG flow is from the UV theory given by

$$Y_{1,0} = [2,2,2]\ , \quad Y_{-1} = [2,2,1,1]\ . \tag{8}$$

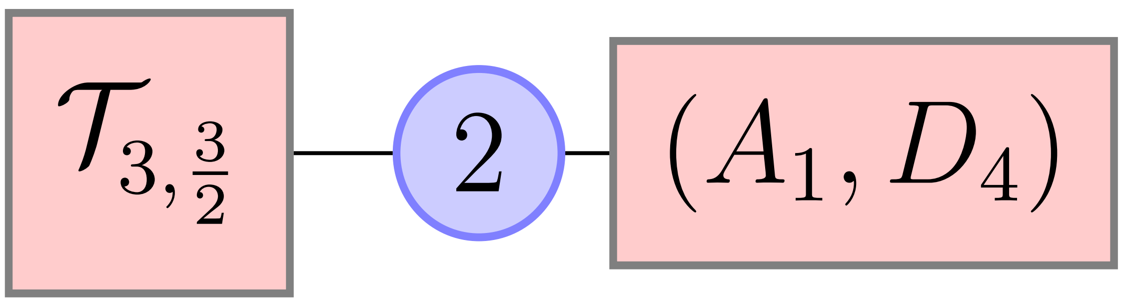
- This description is abstract. Also obtain from duality involving well-known SCFTs [M.B., Giacomelli, Nishinaka, Papageorgakis]

- Start with the following $su(3)$ theory



The Simplest RG Flow (cont...)

- Going to infinite coupling in exactly marginal $\tau_{su(3)}$ leads to a dual "weakly coupled" $su(2)$ description



- The $\mathcal{T}_{3,\frac{3}{2}}$ SCFT is another name for the theory with the Young diagrams in (8). It has $\mathcal{N}=2$ chiral ring generators of dimensions 3 and 3/2 (so no 4D $\mathcal{N}=2$ Lagrangian).

The Simplest RG Flow (cont...)

• This latter theory is subtle, it splits as [M.B., Laczko, Nishinaka]

$$\boxed{\mathcal{T}_{3,\frac{3}{2}}} = \boxed{1} \oplus \boxed{\mathcal{T}_X},$$

where $\mathcal{T}_X$ is non-Lagrangian with flavor symmetry $su(2) \times su(3)$ (so that $\mathcal{T}_{3,\frac{3}{2}}$ has $su(2)^2 \times su(3)$ flavor symmetry).

• First connection with $\mathcal{N} = 4$: the $su(2)$ flavor symmetry in $\mathcal{T}_X$ has a global Witten anomaly

The Simplest RG Flow (cont...)

• More interesting connections come from noting Schur index of $\mathcal{T}_X$ [M.B., Laczko, Nishinaka]

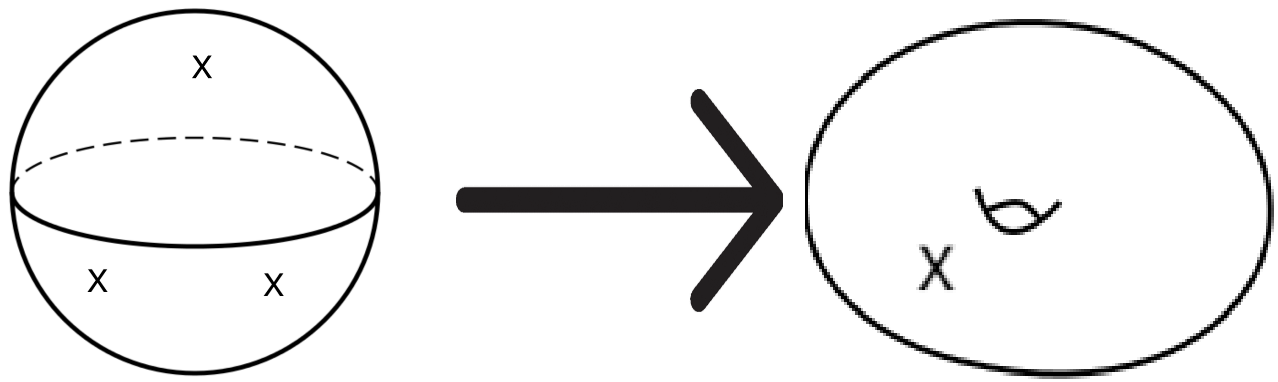
$$\mathcal{I}_{\mathcal{T}_X} = \sum_{\lambda=0}^{\infty} q^{\frac{3}{2}\lambda}\text{P.E.}\left[\frac{2q^2}{1-q}+2q-2q^{1+\lambda}\right]\text{ch}_{R_\lambda}^{su(2)}(q,w)\times \times\text{ch}_{R_{\lambda,\lambda}}^{su(3)}(q,z_1,z_2)\ . \quad (9)$$

• This index is closely related to that of the T_2 theory

$$\mathcal{I}_{T_2} = \sum_{\lambda=0}^{\infty} q^{\frac{\lambda}{2}}\ \text{P.E.}\left[\frac{2q^2}{1-q}+2q-2q^{1+\lambda}\right]\text{ch}_{R_\lambda}^{su(2)}(q,x)\times \times\text{ch}_{R_\lambda}^{su(2)}(q,y)\text{ch}_{R_\lambda}^{su(2)}(q,z)\ , \quad (10)$$

The Simplest RG Flow (cont...)

• Indeed, we can gauge a diagonal $su(2)^2 \subset su(2)^3 \subset sp(4)$ to get $\mathcal{N} = 4$ (plus a decoupled hyper)



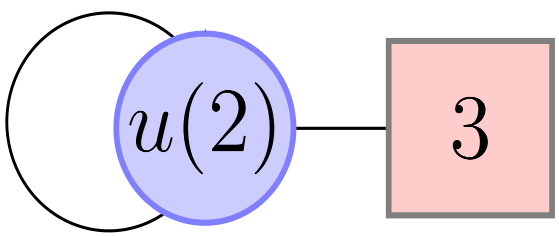
• Might then guess that we should do something involving a vector multiplet of $su(3)$ to get $\mathcal{N} = 4$ from $\mathcal{T}_{3,\frac{3}{2}}$.

The Simplest RG Flow (cont...)

- To see how to proceed, we take the limit

$$\lim_{q\to 1} \mathcal{I}_{\mathcal{T}_{3,\frac{3}{2}}} \sim Z_{S^3} \tag{11}$$

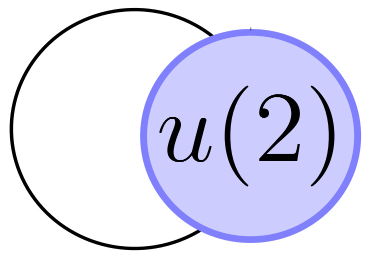
with S^3 partition function for the following [M.B., Laczko, Nishinaka]



where loop is in the $\mathbf{3}+\mathbf{1}$ of $su(2) \subset u(2)$.

The Simplest RG Flow (cont...)

- Turning on $su(3)$ masses yields the following in the IR

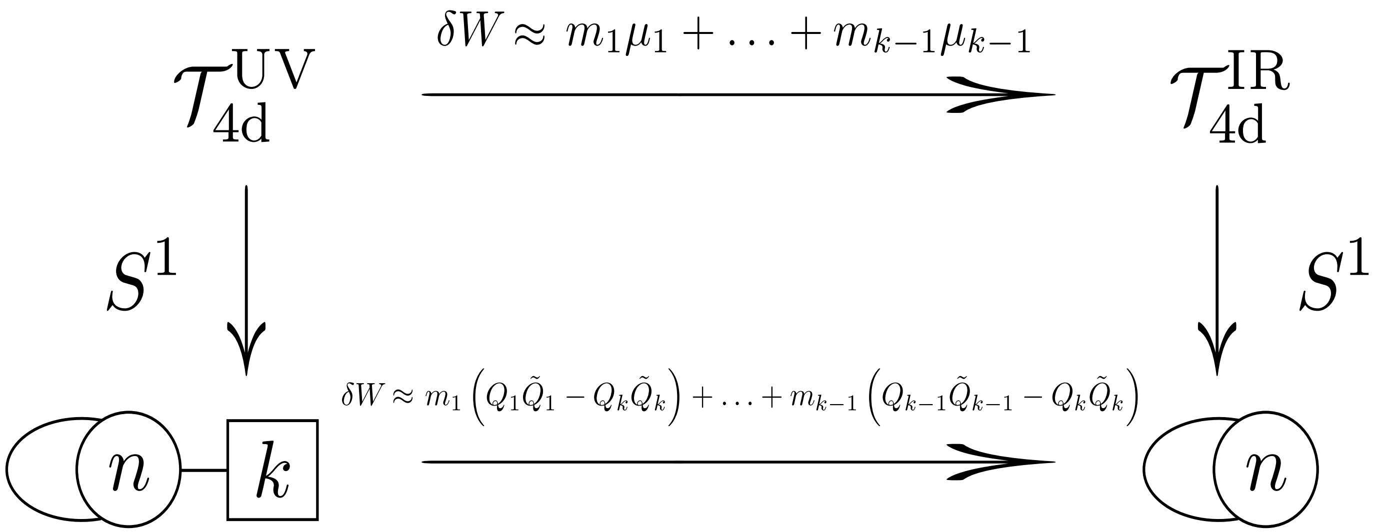


i.e., we flow to the same 3D $\mathcal{N} = 8$ SCFT as the $u(2)$ 3D $\mathcal{N} = 8$ gauge theory.

- This is the dimensional reduction of 4D $\mathcal{N} = 4$ $u(2)$ SYM

The Simplest RG Flow (cont...)

• In particular, we have the following commuting diagram (for $n = 2$ and $k = 3$):



• Each arrow preserves 8 Poincaré supercharges. Commutation of above diagram strongly rests on this fact.

The Simplest RG Flow (cont...)

• What happens in the $r \to \infty$ limit? In particular:

(i) Is the IR theory in the $r \to \infty$ limit 4D $\mathcal{N} = 4$?

(ii) Is it $u(2)$ $\mathcal{N} = 4$ SYM?

• The deformations make it clear that $\mathcal{T}_X$ gives rise to an IR theory with a Witten anomaly for $su(2)$.

• The existence of a 3D Lagrangian doesn't answer these questions.

• To gain some understanding, let's write down the SW curve.

The Simplest RG Flow (cont...)

- In this case, we have

$$\begin{aligned}\Phi_z &= z\ \mathrm{diag}(a_1, a_1, a_2, a_2, -a_1-a_2, -a_1-a_2)\\ &+ \mathrm{diag}(b_1, b_1, b_2, b_2, -b_1-b_2, -b_1-b_2)\\ &+ \frac{1}{z}\ \mathrm{diag}(m_1, m_1, m_2, m_2, -m_1-m_2+m_3, -m_1-m_2-m_3)\\ &+ \frac{1}{z^2}\ \mathrm{diag}(c_1, c_2, c_3, c_4, c_5, -c_1-c_2-c_3-c_4-c_5) + \mathcal{O}(z^{-3}) \quad (12)\end{aligned}$$

- We can compute the SW curve from the spectral curve

$$\det(x - \Phi_z) = 0\ . \quad (13)$$

The Simplest RG Flow (cont...)

- The SW curve is then

$$
\begin{aligned}
u_2 &+ ((x-a_1z)(x-a_2z)(x+(a_1+a_2)z)+\frac{M_1}{2}(x-a_1z) \\
&+ \frac{M_2}{2}(x-a_2z)-b(x-a_1z)(x-a_2z))^2+M_3^2(x-a_1z)(x-a_2z) \\
&+ u_1(-b(a_1-a_2)(x-a_1z)(x-a_2z) \\
&- (x-a_1z)^2(x-a_2z)(a_1+2a_2)+(x-a_1z)(x-a_2z)^2(2a_1+a_2) \\
&+ \frac{a_1-a_2}{2}(M_1(x-a_1z)+M_2(x-a_2z)))=0 \ . \qquad (14)
\end{aligned}
$$

where

$$
\begin{aligned}
M_1 &= -2(a_1+2a_2)m_2 \ , \quad M_2=-2(2a_1+a_2)m_1 \ , \\
M_3^2 &= -(2a_1+a_2)(a_1+2a_2)m_3^2 \ , u_1=-(2a_1+a_2)(c_1+c_2)+2bm_1 \ , \\
u_2 &= (a_1-a_2)^2((2a_1+a_2)c_1-bm_1)((2a_1+a_2)c_2-bm_1) \\
&+ (a_1-a_2)(2a_1+a_2)(a_1+2a_2)m_1m_3^2 \ . \qquad (15)
\end{aligned}
$$

The Simplest RG Flow (cont...)

• This describes the curve of $\mathcal{T}_{3,\frac{3}{2}}$. To get the curve of the IR theory, we introduce an RG parameter, m, and take a scaling limit $m \to \infty$.

• The only consistent scaling limit we have been able to find (up to isomorphisms) is

$$\begin{aligned} u_1 &= 0 , \quad M_1 = m , \quad M_2 = 0 , \quad M_3 = 0 , \quad b = qm^{\frac{1}{2}} , \\ x - a_1 z &= 2m^{-\frac{1}{2}} X , \quad x - a_2 z = m^{\frac{1}{2}} Z , \quad u_2 = -Um . \end{aligned} \tag{16}$$

• Here, X and Z are good coordinates for the curve of the IR theory.

The Simplest RG Flow (cont...)

- Taking this limit yields

$$X^2 = \frac{U}{\left(\frac{2(2a_1+a_2)}{a_1-a_2}Z^2 - 2qZ + 1\right)^2} \ . \qquad (17)$$

- This is the $su(2)$ $\mathcal{N} = 4$ curve tuned to a cusp.

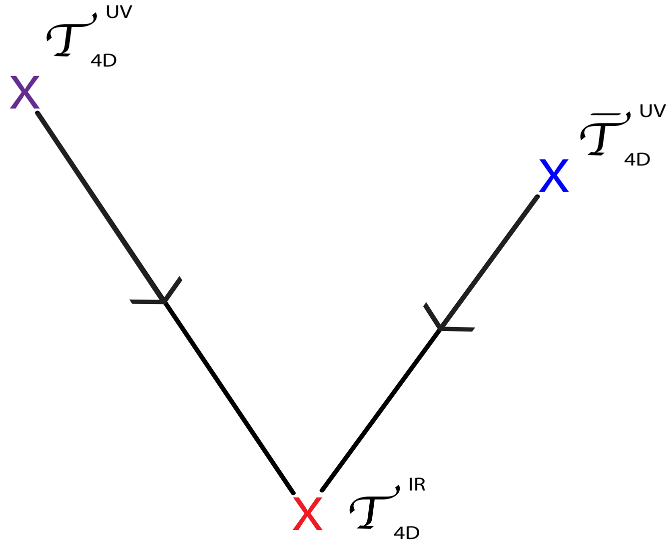
- Note that the putative marginal deformation, $q = bm^{-\frac{1}{2}}$, is irrelevant in the IR.

- These are the most general scaling limits we found. What does this mean?

The Simplest RG Flow (cont...)

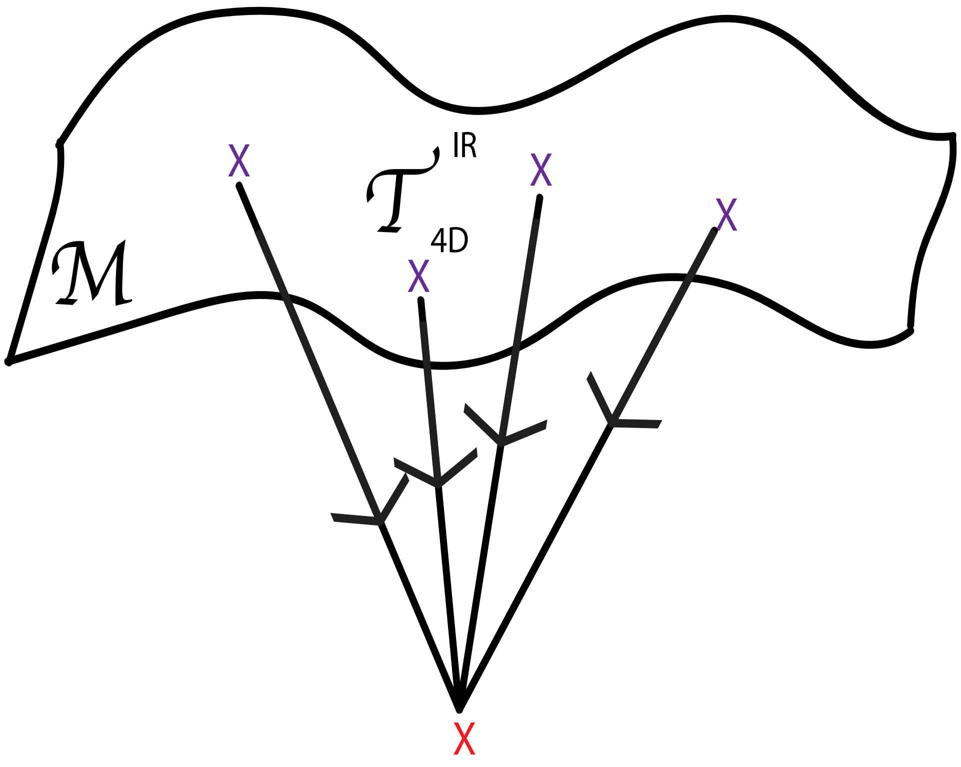
• We haven't been able to disprove that there might be a more general scaling limit at play

• Another possibility is that the IR exactly marginal direction is only visible from a different UV starting point



The Simplest RG Flow (cont...)

- A final possibility is to have an exotic $\mathcal{N} = 4$ theory in IR



Generalizations

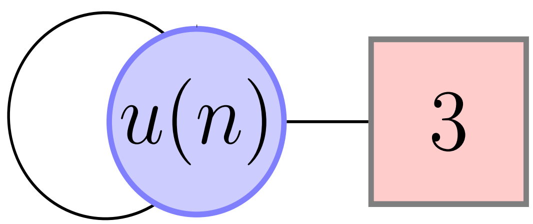
• We can generalize the above picture considerably. For instance, take

$$Y_1 = Y_0 = [n, n, n] \ , \quad Y_{-1} = [n, n, n-1, 1] \ . \tag{18}$$

Also gives 4D SCFT with $\mathcal{N} = 2$ chiral ring generators of dim

$$\Delta = \left\{ \frac{3}{2} \ , \ 3 \ , \ \frac{9}{2} \ , \ \cdots \ , \ \frac{3n}{2} \right\} \ . \tag{19}$$

and S^1 reduction



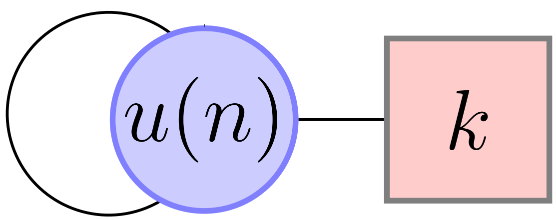
Generalizations (cont...)

- Even more generally, can take

$$Y_{1,0} = [n, \cdots, n] \ , \quad Y_{-1} = [n, \cdots, n, n-1, 1] \ , \qquad (20)$$

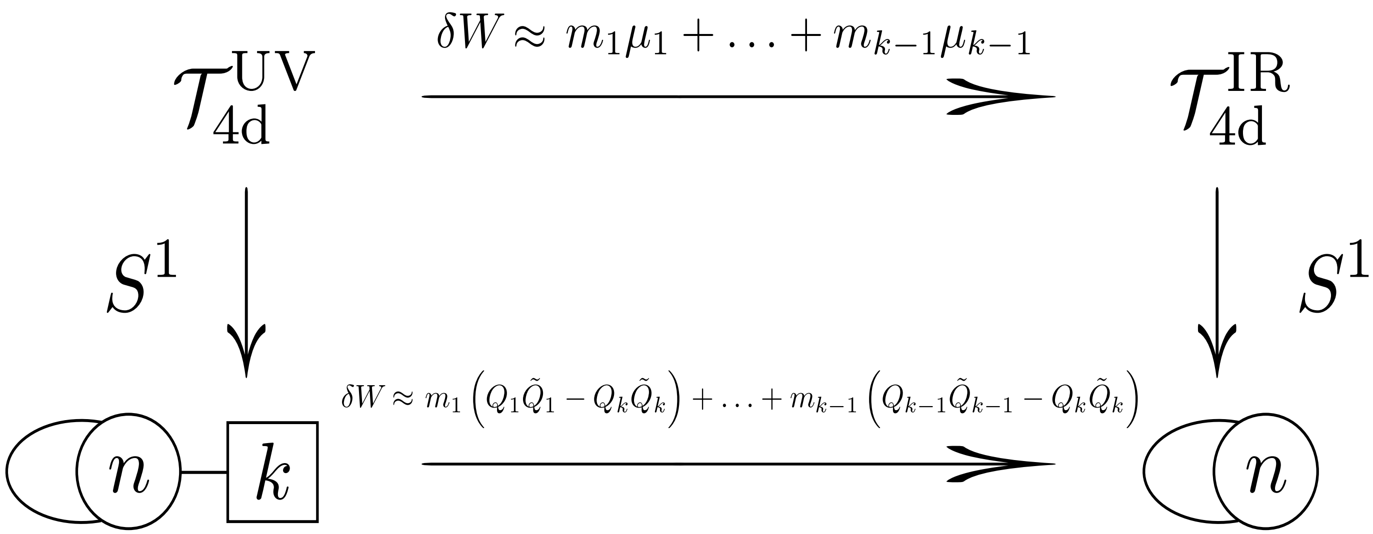
where $n \geq 2$, $k_1 = k_0 \geq 3$ and $k_{-1} = k_1 + 1 \geq 4$ and so $N = nk$.

- Now have



Generalizations (cont...)

- Similar story with dimensional reduction and mass terms



Conclusions and Open Questions

• Have found a new playground for engineering accidental SUSY enhancement.

• At least in $n = 2, k = 3$ case seem to have constructed a 4D $\mathcal{N} = 4$ SCFT without ever discussing free fields.

• What is the nature of this 4D theory?

• Do we know everything there is to know about 4D $\mathcal{N} = 4$? What about 6D $(2, 0)$, etc.?