4d $\mathcal{N} = 3$ indices via discrete gauging

based on T. Bourton, AP and E. Pomoni arXiv:1804.05396 [hep-th]

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Introduction and motivations

- We did not know that $4d$ $\mathcal{N} = 3$ QFTs exist until very recently !!!
  [I.García-Etxebarria and D.Regalado (2015)]
- By $\mathcal{N} = 3$ SCA alone !! $\Rightarrow$ **Main properties**: [O.Aharony and M. Evtikhiev (2015)]
  - Have no marginal couplings $\Rightarrow$ are isolated fixed points.
  - They do not have a Lagrangian description.
  - The only flavour symmetry is an R-symmetry.
  - The conformal central charges (a, c) must be equal.

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Which tools we can use ?

- F-theory and M-theory construction. [I.García-Etxebarria and D.Regalado (2015)]
- Gravity dual description $\Rightarrow$ large N limit. [I.García-Extebarria and D.Regalado (2015)],[O.Aharony and Y.Tachikawa (2016)],[Y.Imamura and S.Yokoyama (2016)]
- Superconformal bootstrap and chiral algebra techniques. [T.Nishinaka and Y.Tachikawa. (2016)],[M.Lemos, P.Liendo et al. (2016)]

Extra tool: Superconformal Index (SCI)

$\Downarrow$

Prescription:

Discrete gauging of $4d$ $\mathcal{N} = 4$ SCI $\Rightarrow$ $4d$ $\mathcal{N} = 3$ SCI

Contents of the talk

1. Constructing 4d $\mathcal{N} = 3$ theories
2. The discrete gauging prescription
3. Results
4. Summary and conclusions

Contents of the talk

Construction via S-folds

[I.García-Etxebarria and D.Regalado (2015)]

F-theory on $\mathbb{R}^{1,3} \times (\mathbb{R}^6 \times T^2)/\mathbb{Z}_k$, $\quad \mathbb{Z}_k \subset \boxed{SO(6)_R} \times \boxed{SL(2,\mathbb{Z})}$.

$$e^{\frac{2\pi i}{k}(r_k + s_k)} \in \mathbb{Z}_k$$

- $\boxed{SO(6)_R}$ r_k acts on $\mathbb{R}^6$
- $\boxed{SL(2,\mathbb{Z})}$ s_k acts on T^2 $\quad z = x + \tau y$
  - For $k = 1, 2 \Rightarrow \quad \mathcal{N} = 4$ SYM $\quad (k = 2 \ \rightarrow$ orientifolds).
  - For $k = 3, 4, 6 \Rightarrow$ **fixed** complex structure $\tau = e^{i\pi/3}, i, e^{i\pi/3}$.

$\mathcal{N} = 3$ **strongly coupled SCFT**

4d $\mathcal{N} = 3$ theories via discrete gauging

[O.Aharony and Y.Tachikawa (2016)], [P.C.Argyres and M.Martone (2018)]

Mother theory: $\mathcal{N} = 4$ SYM in $4d$ (G, τ). $\mathfrak{g} = \mathrm{Lie}(G) = A, D, E$ or $\mathfrak{u}(N)$.

We focus on local operators $\Rightarrow SL(2, \mathbb{Z})$ is the self duality group.

The global symmetry group is at least: $PSU(2, 2 \,|\, 4)$

If $\tau = \{\mathrm{any}, e^{i\pi/3}, i, e^{i\pi/3}\} \Rightarrow$ $\mathbb{Z}_n$ enhancement $n = 2, 3, 4, 6$

$\mathbb{Z}_n$ $\Downarrow$ gauging

Daughter theory: $\mathcal{N} = 4$, $n = 2$ or $\mathcal{N} = 3$, $n = 3,4,6$

4d $\mathcal{N}=3$ as $\mathcal{N}=2$ QFTs

$U(3)_R \mapsto SU(2)_R \times U(1)_r \times U(1)_f$ $\qquad (X, Y, Z) \in V_{\mathcal{N}=4}$

Coulomb branch CB $(Y = Z = 0)$

$$X \mapsto u_j : \quad 1 \leq j \leq N = \mathrm{rank}[\mathfrak{g}], \quad E(u_j) = r(u_j),$$

$$\boxed{\mathbb{Z}_n : u_j \mapsto e^{\frac{2\pi i}{n} E(u_j)} u_j}$$

Higgs branch HB $(X = 0)$

$$(Y, Z) \mapsto W^{(f)} : \quad E(W^{(f)}) = 2R(W^{(f)}) \quad f \in U(1)_f$$

$$\boxed{\mathbb{Z}_n : W^{(f)} \mapsto e^{\frac{2\pi i}{n} f} W^{(f)}}$$

Contents of the talk

Computation of the $\mathcal{N} = 4$ mother theory SCI

[C. Romelsberger (2005)], [J.Kinney, J.M.Maldacena, S.Minwalla and S.Raju (2005)]

$$\mathcal{I}(\mu_i) = \mathrm{Tr}_{\mathbb{S}^3}\left[(-1)^F e^{\mu_i J_i}\right], \quad [J_i, \mathcal{Q}] = 0 \quad \delta = \{\mathcal{Q}, \mathcal{Q}^\dagger\} = 0$$

$$\frac{\partial}{\partial\tau}\mathcal{I}(\mu_i) = 0 \quad \Rightarrow$$

Computation in the free theory

$$\int d\mu_G \mathrm{PE}[\boxed{i(\ldots)}\chi^G_{adj}]$$

Single letter index

The index gauging prescription

[Thomas Bourton, AP and E.Pomoni (2018)]

At $\tau = \{\mathrm{any}, e^{\pi i/3}, i, e^{\pi i/3}\} \Rightarrow \mathbb{Z}_n$ enhancement $\Rightarrow$ **extra fugacity ϵ**

$$\mathcal{I}_{\mathcal{M}_{\mathcal{N}=4}}(\mu_i, \epsilon) = \sum_{\mathcal{M}_{\mathcal{N}=4} \in shorts} \mathcal{I}_{\mathcal{M}_{\mathcal{N}=4}}(\mu_i) \epsilon^{r_n + s_n}$$

We gauge the $\mathbb{Z}_n$ symmetry

$$\mathcal{I}_{\mathbb{Z}_n}(\mu_i) := \frac{1}{|\mathbb{Z}_n|} \sum_{\epsilon \in \mathbb{Z}_n} \mathcal{I}_{\mathcal{M}_{\mathcal{N}=4}}(\mu_i, \epsilon) \quad \Rightarrow \quad r_n + s_n = 0 \mod n$$

Projection to $\mathbb{Z}_n$ gauge invariant operators

In these two cases there is not recombination !!!

[A.Gadde, L.Rastelli, S.S.Razamat and W.Yan (2011)]

The Coulomb branch limit: $X \quad (Y, Z = 0)$

$$i_{\mathrm{CB}}(x) = x, \quad (r_n + s_n)(X) = r \quad \Rightarrow \quad \boxed{x \mapsto \epsilon x}$$

$$\mathcal{I}_{\mathbb{Z}_n,\mathrm{CB}}(x) = \frac{1}{|\mathbb{Z}_n|}\sum_{\epsilon\in\mathbb{Z}_n}\int d\mu_G \mathrm{PE}[i_{\mathrm{CB}}(x\epsilon)\chi^G_{adj}]$$

Higgs branch Hilbert series: $(Y, Z), \quad (X = 0)$

$$\mathrm{HS}(\mathfrak{t}, y, \epsilon) := \mathrm{Tr}_{\mathcal{H}}[\mathfrak{t}^{2R} y^f \epsilon^{r_n+s_n}], \quad (r_n + s_n)(Y, Z) = f \quad \Rightarrow \quad \boxed{y \mapsto \epsilon y}$$

$$\mathrm{HS}_{\mathbb{Z}_n}(\mathfrak{t}, y) = \frac{1}{|\mathbb{Z}_n|}\sum_{\epsilon\in\mathbb{Z}_n}\int d\mu_G \mathcal{F}(\mathfrak{t}, \epsilon y)$$

Contents of the talk

Coulomb branch

$$\mathcal{I}^{\mathfrak{u}(1)}_{\mathbb{Z}_n,\mathrm{CB}}(x) = \mathrm{PE}[x^n], \qquad \mathrm{CB}_{\mathfrak{u}(1),n} \cong \mathbb{C}$$

$$u = X \ \mapsto \ \widetilde{u} = u^n,$$

agrees with [O.Aharony and Y.Tachikawa (2015)], [P.C.Argyres and M.Martone (2016)]

Higgs branch

$$\mathrm{HS}^{\mathfrak{u}(1)}_{\mathbb{Z}_n}(\mathfrak{t}, y) = \mathrm{PE}[\mathfrak{t}^2 + (y^n + y^{-n})\mathfrak{t}^n - \mathfrak{t}^{2n}], \quad \mathrm{HB}_{\mathfrak{u}(1),n} = \mathbb{C}^2/\mathbb{Z}_n,$$

$$J = YZ, \quad W^+ = Y^n, \quad W^- = Z^n, \qquad W^+W^- = J^n$$

agrees with [T.Nishinaka and Y.Tachikawa (2016)]

Rank-1 theories: $G = SU(2)$

[Thomas Bourton, AP and E.Pomoni (2018)]

$$\mathcal{I}^{\mathfrak{su}(2)}_{\mathbb{Z}_n,\mathrm{CB}}(x) = \begin{cases} \mathrm{PE}[x^2] & \text{n=1} \\ \mathrm{PE}[x^n] & \text{n=2,4,6} \\ \mathrm{PE}[x^6] & \text{n=3} \end{cases} \qquad \mathrm{CB}_{\mathfrak{su}(2),n} \cong \mathbb{C}$$

$$u = \frac{1}{2}\mathrm{Tr}[X^2] \ \mapsto\ \widetilde{u} = u^{n/E(u)} \ (n = 2, 4, 6) \qquad u \mapsto \widetilde{u} = u^3 \ (n = 3)$$

agrees with [P.C.Argyres and M.Martone (2016)]

$$\mathrm{HS}^{\mathfrak{su}(2)}_{\mathbb{Z}_n}(\mathfrak{t}, y) = \begin{cases} \mathrm{PE}[(1 + y^2 + y^{-2})\mathfrak{t}^2 - \mathfrak{t}^4] & \text{n=1} \\ \mathrm{PE}[\mathfrak{t}^2 + (y^n + y^{-n})\mathfrak{t}^n - \mathfrak{t}^{2n}] & \text{n=2,4,6} \\ \mathrm{PE}[\mathfrak{t}^2 + (y^6 + y^{-6})\mathfrak{t}^6 - \mathfrak{t}^{12}] & \text{n=3} \end{cases} \qquad \mathrm{HB}_{\mathfrak{su}(2),n} \cong \mathbb{C}^2/\mathbb{Z}_n \ (n = 2, 4, 6)$$

$$W^+ = \frac{1}{2}\mathrm{Tr}[Y^n], \quad W^- = \frac{1}{2}\mathrm{Tr}[Z^n], \quad J = \frac{1}{2}\mathrm{Tr}[YZ] \qquad W^+W^- = J^n$$

agrees with [T.Nishinaka and Y.Tachikawa (2016)]

Higher ranks Coulomb branches

[T.Bourton, AP and E.Pomoni (2018)]

E.g. $\mathfrak{g} = A_3$ $\quad u_j = \frac{1}{j}\mathrm{Tr}[X^j], \quad u_j \mapsto e^{\frac{2\pi i}{n}\Delta(u_j)} u_j, \quad j = 2, 3, 4$

n	$\mathrm{PLog}\left[\mathcal{I}^{\mathfrak{su}(4)}_{\mathbb{Z}_n,\mathrm{CB}}(x)\right]$	Generators	Relation
1	$x^2 + x^3 + x^4$	$u_2,\ u_3,\ u_4$	freely generated
2	$x^2 + x^4 + x^6$	$u_2,\ u_4,\ u_3^2$	freely generated
3	$x^3 + 2x^6 + x^{12} - x^{18}$	$u_3,\ u_2^3,\ u_2u_4,\ u_4^3$	$u_2^3u_4^3 = (u_2u_4)^3$
4	$2x^4 + x^8 + x^{12} - x^{16}$	$u_2^2,\ u_4,\ u_2u_3^2,\ u_3^4$	$u_2^2u_3^4 = (u_2u_3^2)^2$
6	$3x^6 + x^{12} - x^{18}$	$u_2^3,\ u_3^2,\ u_2u_4,\ u_4^3$	$u_2^3u_4^3 = (u_2u_4)^3$

Generically **higher ranks CBs are not freely generated !!!!**

agrees with [P.C. Argyres and M.Martone (2018)], [A.Bourget, AP and D.Rodríguez-Gómez (2018)]

Contents of the talk

Summary and conclusions

We gave the prescription for the computation of the SCI

$\boxed{\begin{array}{c} 4d\ \ \mathcal{N}=4\ \ (G,\tau) \\ \mathbb{Z}_n\ \text{ enhancement}\end{array}}$ $\Rightarrow$ $\mathbb{Z}_n$ gauging $\Rightarrow$ $\boxed{4d\ \mathcal{N}=3\ \text{QFT}}$

- We studied:
  - The Coulomb branch limit.
  - The Higgs branch Hilbert series.
- In general the CBs of the daughter QFTs are not freely generated.

THANKS FOR YOUR ATTENTION

$su(2,2|\mathcal{N})$ representation theory

- $su(2,2|4)$ $(E, j_1, j_2, R_1, R_2, R_3)$, the maximal bosonic subalgebra reads

$$u(1)_E \oplus su(2)_1 \oplus su(2)_2 \oplus su(4)$$

$$su(4) \to su(3) \oplus u(1)_{r_{\mathcal{N}=3}}$$

- $su(2,2|3)$ $(E, j_1, j_2, R_1, R_2, r_{\mathcal{N}=3})$, the maximal bosonic subalgebra

$$u(1)_E \oplus SU(2)_1 \oplus SU(2)_2 \oplus su(3) \oplus u(1)_{r_{\mathcal{N}=3}}$$

where

$$r_{\mathcal{N}=3} = \frac{R_1}{3} + \frac{2R_2}{3} + R_3$$

$su(2,2|\mathcal{N})$ representation theory

$$su(3) \oplus u(1)_{r_{\mathcal{N}=3}} \to su(2)_R \oplus u(1)_r \oplus u(1)_f$$

We choose $su(2,2|2) \subset su(2,2|3)$ (E, j_1, j_2, R, r, f)

$$u(1)_E \oplus su(2)_1 \oplus su(2)_2 \oplus su(2)_R \oplus u(1)_r \oplus u(1)_f$$

where

$$r = \frac{R_1}{2} + R_2 + \frac{R_3}{2}, \quad R = \frac{R_1}{2}, \quad f = R_3$$

S-fold construction

[O.Aharony and Y.Tachikawa (2016)]

S-folds, labelled by $\mathbb{Z}_p \subset \mathbb{Z}_k$ global symmetries

Mother Theory: $(N, k, l = \frac{k}{p} \in \mathbb{Z})$

Coulomb branch operators MT: $k, 2k, \ldots, (N-1)k; Nl$

Discrete gauging $\mathbb{Z}_p$: **MT** $\longrightarrow$ **Daughter Theory** (N, k, l, p)

Coulomb branch operators DT: $k, 2k, \ldots, (N-1)k; Np$

$\Rightarrow$ **No relations !!!**