

# The Importance of Being Disconnected: Principal Extension Gauge Theories

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Work with Alessandro Pini and Diego Rodriguez-Gomez (1804.01108)

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### INTRODUCTION

Gauge theories initially formulated using simple Lie algebras. Possible extensions include:

Products of many Lie algebras / groups



[Gaiotto, 0904.2715]

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#### INTRODUCTION

Gauge theories initially formulated using simple Lie algebras. Possible extensions include:

Products of many Lie algebras / groups



[Gaiotto, 0904.2715]

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• Global structure  $(\pi_1)$  of the group



Figure 1: The weights of line operators of gauge theories with  $\mathbf{g} = su(2)$ .

[Aharony, Seiberg, Tachikawa, 1305.0318]

• What about the global structure  $\pi_0$  of the group?

Recent interest on discrete gauging (recall Hanany's talk). Gauging of a discrete symmetry allows for new Coulomb branch geometries

Discrete gauge group action on the Coulomb Branch:										CFT data:			
parent	$\mathbb{Z}_2$	Ž2	Z <sub>8</sub>	Ža	$\mathbb{Z}_4$	Ž4	$\mathbb{Z}_{5}$	$\mathbb{Z}_4$	2e	$h_F$	240	12c	
[11 <sup>*</sup> ,E <sub>8</sub> ]										12	95	62	0
[111, 187]										8	59	38	0
$[IV^{+},E_{0}]$	$[II^*_{j}F_4]$									6	41	26	0
$[I_0, D_4 X_0]$	[111*, B3]		$[II^*,G_2]$							4	23	14	0
[IV.AsX1]	[11.42]				$[II^*, B_1]$					3	14	8	0
↓ <sup>*</sup> [III],A1X9]	1		1 [111*,A1]							8	11	6	0
1.1											43	22	
[II.x <sub>8</sub> ]	i.		1		1		[11*,0]			-	5	3	0
$[I_0, \mathfrak{S}]$	$[I_0^*, B]$		$[IV_1^T, \vartheta]$		[111*,8]			[11*,0]		-	5	2	0
[11*,C5]										7	82	49	5
$[III^{+}, C_{1}C_{1}]$										(5,8)	50	29	3
$[IV^{*}, C_{2}U_{1}]$	$[II^{*}_{C_{2}}]$									(4,7)	34	19	4
[I_0,C_1X_0]	$[III^{*}, C_{1}]$	$[III^*, U_1 \otimes \mathbb{Z}_2]$								3	18	9	1
[4,01]	$[I_2^*, \sigma]$	[12,0]								1	6	3	0
$[II^*, A_3 \rtimes \mathbb{Z}_2]$										14	75	42	4
111, 110	$z_2$									(10,?	) 45	$^{24}$	2
$[IV^*, U_1]$	$[II^+, \theta]$									5	30	15	1
$[J_1^*, \emptyset]$										-	17	8	0
$[II^*, A_2 \times \mathbb{Z}_2]$										14	71	38	3
111 .0	1									7	42	$^{21}$	1
$[IV_{1}^{+}, a]$										-	55	$\frac{25}{2}$	0
$[I_0^*, C_1 \chi_0]$	$[HI^{\bullet}, C_{1}]$	$[III^*,U_1\times\mathbb{Z}_2]$	$[II^*,C_1]$	$(H^*, U_1 \rtimes \mathbb{Z}_2)$						3	18	9	1
[J2,U1]	[11,0]	[I_1, 0]		i.						1	6	3	0
[4, 0]			$[IV_{\sqrt{2}}^{+}, \theta]$	$[IV_{\sqrt{2}}^{+}, \theta]$						-	5	2	0
$I_0.C_1 x_0]$	$[I_0^*, x_0] \times H$	$[I_0^*, C_1 x_0]$	$[IV_1^*, \vartheta] \times \mathbb{H}$	[IV-3/1	[111*,Ø]×H	(m, n)	ia]	[11*,0]×H	$[II^*,U_1\!\!\times\!\! \mathbf{Z}_2]$	1	6	3	1
[4.9]	125.21	[15.9]	[[V1.0]	[[V_1],g]	1111 .01	1111 . 11		111.0	[11".0]	-	5	2	0

[Argyres, Martone, 1611.08602]

Alternative approach : start from a disconnected gauge group.

In this work

- We consider a special class of non-connected groups
- ▶ We focus on 4d  $\mathcal{N} = 2$  SCFTs
- We look at local physics and use algebraic counting tools.

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The Coulomb branch Index

The Higgs Branch of SQCD and the Global Symmetry

THE COULOMB BRANCH INDEX

The Higgs Branch of SQCD and the Global Symmetry



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Definition of principal extension

 $\widetilde{G} = \mathit{G}_{\mathrm{adj}} \rtimes_{\varphi} \mathsf{\Gamma}_{\mathrm{outer}}$ 

Examples :

$$\widetilde{\mathrm{SU}}(N) = \mathrm{SU}(N) \rtimes_{\varphi} \{1, \mathcal{P}\}$$

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Examples :

$$\widetilde{\mathrm{SU}}(N) = \mathrm{SU}(N) \rtimes_{\varphi} \{1, \mathcal{P}\}$$

$$\widetilde{\mathrm{SO}}(2N) = \mathrm{SO}(2N) \rtimes_{\varphi} \{1, \mathcal{P}\} = \mathrm{O}(2N)$$

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Examples :



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Examples :



 $(X,1) \qquad (X,\mathcal{P})$ 

In the case of SU(N), this is related to complex conjugation:

$$\overline{X} = A^{-1} \mathcal{P}(X) A$$
,  $A^T = (-1)^{N-1} A$  and  $\det A = 1$ .

# Representations $(\widetilde{\mathrm{SU}}(3) \text{ example})$



### The Coulomb branch Index

## The Higgs Branch of SQCD and the Global Symmetry

Recall the superconformal index for a theory with gauge group G and some fundamental matter multiplets:

$$\mathcal{I} = \int \mathrm{d}\eta_{\mathcal{G}}(X) \operatorname{PE}\Big[\sum_{i \in \mathrm{multiplets}} f^{\mathcal{R}_i} \chi_{\mathcal{R}_i}(X)\Big];$$

where

$$f^{V} = -\frac{\sigma \tau}{1 - \sigma \tau} - \frac{\rho \tau}{1 - \rho \tau} + \frac{\sigma \rho - \tau^{2}}{(1 - \rho \tau)(1 - \sigma \tau)},$$
$$f^{\frac{1}{2}H} = \frac{\tau (1 - \rho \sigma)}{(1 - \rho \tau)(1 - \sigma \tau)}.$$

In the limit

$$au 
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ho \sigma =: t.$$

we have

$$f^V = t, \qquad f^{\frac{1}{2}H} = 0.$$

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This gives the Coulomb branch index [Gadde, Rastelli, Razamat, 1110.3740].

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$$\mathcal{I}_{G}^{\mathrm{Coulomb}}(t) = \int_{G} \mathrm{d}\eta_{G}(X) \frac{1}{\det\left(1 - t \Phi_{\mathrm{Adj}}(X)\right)}$$

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This is Molien's formula for the Hilbert series of invariants of the adjoint representation.

# WHAT IS A HILBERT SERIES

Let *R* be a Noetherian graded ring with  $R_0 = \mathbb{C}$ ,

$$R = \bigoplus_{n \in \mathbb{N}} R_n$$

The Hilbert series of R is

$$\mathrm{HS}(R,t)=\sum_{n\in\mathbb{N}}t^{n}\dim_{\mathbb{C}}R_{n}.$$

Example :

$$\operatorname{HS}\left(\mathbb{C}[x],t
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Example :

$$\operatorname{HS}\left(\mathbb{C}[x],t\right) = \frac{1}{1-t}$$

For polynomial rings, the Hilbert series is always a rational function [Hilbert, 1890.xxxx]

If the ring is a *complete intersection*,

$$\mathrm{HS}\left(\mathbb{C}[\mathrm{Gens}]/(\mathrm{Rels}),t\right) = \frac{\prod\limits_{\mathrm{Rels}} \left(1 - t^{\mathsf{deg}(\mathrm{Rels})}\right)}{\prod\limits_{\mathrm{Gens}} \left(1 - t^{\mathsf{deg}(\mathrm{Gens})}\right)}$$

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Example:

$$\mathrm{HS}\left(\mathbb{C}[x_1, x_2, x_3]/(x_1x_2 - x_3^2), t\right) = 1 + 3t + 5t^2 + ... = \frac{1 - t^2}{(1 - t)^3}$$

## BASIC INVARIANT THEORY

Particular case: freely-generated ring

$$\mathbb{C}[\mathbf{x}]^G \cong \mathbb{C}[I_1,\ldots,I_m], \qquad HS(\mathbb{C}[\mathbf{x}]^G,t) = \frac{1}{\prod\limits_{i=1}^m (1-t^{\deg I_i})}.$$

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In general, Hironaka decomposition for invariant rings:

$$\mathbb{C}[\mathbf{x}]^G \cong igoplus_{j=1}^p J_j \mathbb{C}[I_1, \dots, I_m]$$
 $HS(\mathbb{C}[\mathbf{x}]^G, t) = rac{\sum\limits_{j=1}^p t^{\deg J_j}}{\prod\limits_{i=1}^m (1 - t^{\deg I_i})}.$ 

Consider a finite group G, and a (finite-dimensional complex) representation V. Consider the ring of invariants  $\mathbb{C}[V]^{G}.$ 

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Molien's formula:

$$\operatorname{HS}\left(\mathbb{C}[V]^{G}, t\right) = \frac{1}{|G|} \sum_{g \in G} \frac{1}{\operatorname{det}_{V}(1 - t \cdot g)}.$$

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For a large class of groups ("reductive"),

$$\frac{1}{|G|}\sum_{g\in G}\longrightarrow \int_{G}\mathrm{d}\eta_{G}(X)\,.$$

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# Weyl Integration Formula

For a class function f,

$$\int_{\widetilde{\mathrm{SU}}(N)} \mathrm{d}\eta_{\widetilde{\mathrm{SU}}(N)}(X) f(X) = \frac{1}{2} \left[ \int \mathrm{d}\mu_N^+(z) f(z) + \int \mathrm{d}\mu_N^-(z) f(z^{\mathcal{P}}) \right]$$

[Wendt, 1999]

with

$$\mathrm{d}\mu_N^+(z) = \prod_{j=1}^{N-1} \frac{\mathrm{d}z_j}{2\pi i z_j} \prod_{\alpha \in R^+(A_{N-1})} \left(1 - z(\alpha)\right) \,,$$

and

$$N \text{ even:} \qquad \mathrm{d} \mu_N^-(z) = \prod_{j=1}^{N/2} \frac{\mathrm{d} z_j}{2\pi i z_j} \prod_{\alpha \in R^+(B_{N/2})} \left(1-z(\alpha)\right) \,.$$

$$N$$
 odd:  $\mathrm{d}\mu_N^-(z) = \prod_{j=1}^{(N-1)/2} \frac{\mathrm{d}z_j}{2\pi i z_j} \prod_{\alpha \in R^+(C_{(N-1)/2})} (1-z(\alpha)) \; .$ 

Computation for SU(N):

$${\mathcal I}^{
m Coulomb}_{{
m SU}({\mathcal N})}(t) = rac{1}{\prod\limits_{i=2}^{{\mathcal N}}(1-t^i)}\,,$$

corresponds to

$$\mathbb{C}[\phi_{ij}]^{\mathrm{SU}(N)} \cong \mathbb{C}[\mathrm{Tr}(\phi^k)_{k=2,\ldots,N}],$$

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polynomial ring without any relation.

Computation for  $\widetilde{SU}(N)$ :

$$\mathcal{I}^{ ext{Coulomb}}_{ ext{SU}(\mathcal{N})}(t) = rac{\sum\limits_{k_1 < \cdots < k_r ext{ odd}} t^{k_1 + \cdots + k_r}}{\prod\limits_{i ext{ even}} (1-t^i) \prod\limits_{i ext{ odd}} (1-t^{2i})}\,,$$

Why?

$$\operatorname{Tr}((\phi^{\mathcal{P}})^k) = (-1)^k \operatorname{Tr}(\phi^k).$$

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There are "holes" in the structure of invariants.

## The Non-Freely Generated Coulomb Branch

Invariant theory interpretation:

1. The *primary* invariants  $I_k$  for  $2 \le k \le N$  defined by

$$I_k = egin{cases} {
m Tr}(\phi^k) & ext{ for } k ext{ even} \ {
m Tr}(\phi^k)^2 & ext{ for } k ext{ odd} \end{cases}$$

2. The secondary invariants

$$J_{k_1,\ldots,k_r}=\prod_{i=1}^r \operatorname{Tr}(\phi^{k_i}),$$

for  $k_1, \ldots, k_r$  odd and  $3 \le k_1 < \cdots < k_r \le N$ , with r even (r = 0 corresponds to the trivial invariant 1).

Relations (among others):

$$J_{k_1,\ldots,k_r}^2-I_{k_1}\ldots I_{k_r}=0,$$

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For  $N \ge 5$  the Coulomb branch of  $\mathcal{N} = 2$   $\widetilde{\mathrm{SU}}(N)$  gauge theories (of genus 0) is not freely generated.

See also [Argyres, Martone, 1804.03152 ] [Bourton, Pini, Pomoni, 1804.05396 ]

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Example for N = 5: Coulomb branch parametrized by  $I_2$ ,  $I_3$ ,  $I_4$ ,  $I_5$ ,  $J_{3,5}$  with relation  $J_{3,5}^2 = I_3 I_5$ .

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Complex singularity of complex dimension 2 parametrized by  $I_2$  and  $I_4$ .

THE COULOMB BRANCH INDEX

The Higgs Branch of SQCD and the Global Symmetry

## HIGGS BRANCH HILBERT SERIES

The Higgs branch is

 $\left(\mathbb{C}[Q,\tilde{Q}]/(\mathbb{C}\text{F-terms})\right)^{\text{Gauge group}}\,.$ 

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Consider the  $\mathcal{N} = 2$  SU(N) gauge theory with 2N fundamental hypers.

$$W \sim \operatorname{Tr} \tilde{Q} \phi Q \implies Q \tilde{Q} - \frac{1}{N} (\operatorname{Tr} Q \tilde{Q}) \mathbf{1}_N = 0$$

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Higgs branch Hilbert series:

[Hanany, Feng, He, Mekareeya, Benvenuti,...]

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$$H_{\mathrm{SU}(N)} = \int \mathrm{d}\eta_{\mathrm{SU}(N)}(X) \frac{\det\left(1 - t\Phi_{\mathrm{Adj}}(X)\right)}{\det\left(1 - t\Phi_{\mathrm{F}}(X)\right)^{2N}\det\left(1 - t\Phi_{\mathrm{F}}(X)\right)^{2N}} \,.$$

Refine using (S)U(2N) global fugacities. Example

$$H_{\rm SU(3)} = 1 + t^2 \left(\chi_{10001} + 1\right) + 2t^3 \chi_{00100} + t^4 \left(\chi_{01010} + \chi_{10001} + \chi_{20002} + 1\right) + \dots$$

Now  $\widetilde{SU}(N)$ 

$$H_{\widetilde{SU}(N)} = \int \mathrm{d}\eta_{\widetilde{SU}(N)}(X) \frac{\det\left(1 - t^2 \Phi_{\mathrm{Adj}}(X)\right)}{\det\left(1 - t\chi_{10\dots0}^{\mathrm{Flav}} \otimes \Phi_{\mathrm{F\bar{F}}}(X)\right)} \,,$$

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What is the flavor symmetry group?

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What is the flavor symmetry group?

Mesons satisfy symmetry / antisymmetry relations depending on the parity of  $\it N.$ 

## The Higgs Branch of SQCD



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$$\begin{aligned} \mathcal{H}_{SU(3)} &= 1 + 36t^2 + 40t^3 + 630t^4 + 1120t^5 + 7525t^6 + \dots \\ \mathcal{H}_{\widetilde{SU}(3)} &= 1 + 21t^2 + 20t^3 + 336t^4 + 560t^5 + 3850t^6 + \dots \end{aligned}$$

The mesons are symmetric

$$\begin{aligned} \mathcal{H}_{\widetilde{SU}(3)} &= 1 + [2,0,0]_{C_3} t^2 + \left( [0,0,1]_{C_3} + [1,0,0]_{C_3} \right) t^3 \\ &+ \left( 2 [0,1,0]_{C_3} + 2 [0,2,0]_{C_3} + [4,0,0]_{C_3} + 2 \right) t^4 + \dots \end{aligned}$$

$$\begin{split} \mathcal{H}_{SU(4)} &= 1 + 64t^2 + 2156t^4 + 49035t^6 + \dots \\ \mathcal{H}_{\widetilde{SU}(4)} &= 1 + 28t^2 + 1106t^4 + 24381t^6 + \dots \end{split}$$

The mesons are anti-symmetric

$$\begin{aligned} \mathcal{H}_{(4,\,8)} &= 1 + [0,1,0,0]_{D_4} \, t^2 + \left( 2 \, [0,0,0,2]_{D_4} + 2 \, [0,0,2,0]_{D_4} \right. \\ &+ 2 \, [0,2,0,0]_{D_4} + 2 \, [2,0,0,0]_{D_4} + [4,0,0,0]_{D_4} + 2 \right) t^4 \\ &+ \dots \end{aligned}$$

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On a space-time manifold with non-trivial cycles, one can define gauge bundles with disconnected groups. Put the class S theory



on  $\mathcal{C} imes \mathbb{R}^{1,2} imes S^1$  with:

- Twisted punctures
- ► Twist along S<sup>1</sup>.

#### The twisted sector (alone) has been studied [Mekareeya, Song, Tachikawa, 1212.0545]

The index for a sphere with 3 punctures is

$$ilde{l} = \sum_{\lambda} \prod_{i=1,2,3} rac{ ilde{\kappa}_{\Lambda_i}(\mathbf{a}_i) ilde{P}_{\lambda}(\mathbf{a}_i t^{\Lambda_i})}{ ilde{\kappa}_{
ho} ilde{P}_{\lambda}(t^{
ho})}$$

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 $\implies$  "TQFT" structure of superconformal index.

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Work in progress : combined sectors.

- $\blacktriangleright$  Representation theory of principal extension allows to construct interesting Lagrangian  $\mathcal{N}=2~\text{SCFTs}$
- They have non freely generated CBs in general
- ▶ They are type A theories with orthogonal / symplectic global symmetries

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Quivers and brane realization

Spectrum of line operators, other extended operators

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Global anomalies ?

- Spectrum of line operators, other extended operators
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- Compactification on a circle and relation with affine gauge symmetry
- Global anomalies ?

Thank you for your attention!

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