Non-Geometric Calabi-Yau Backgrounds

CH, Israel and Sarti 1710.00853

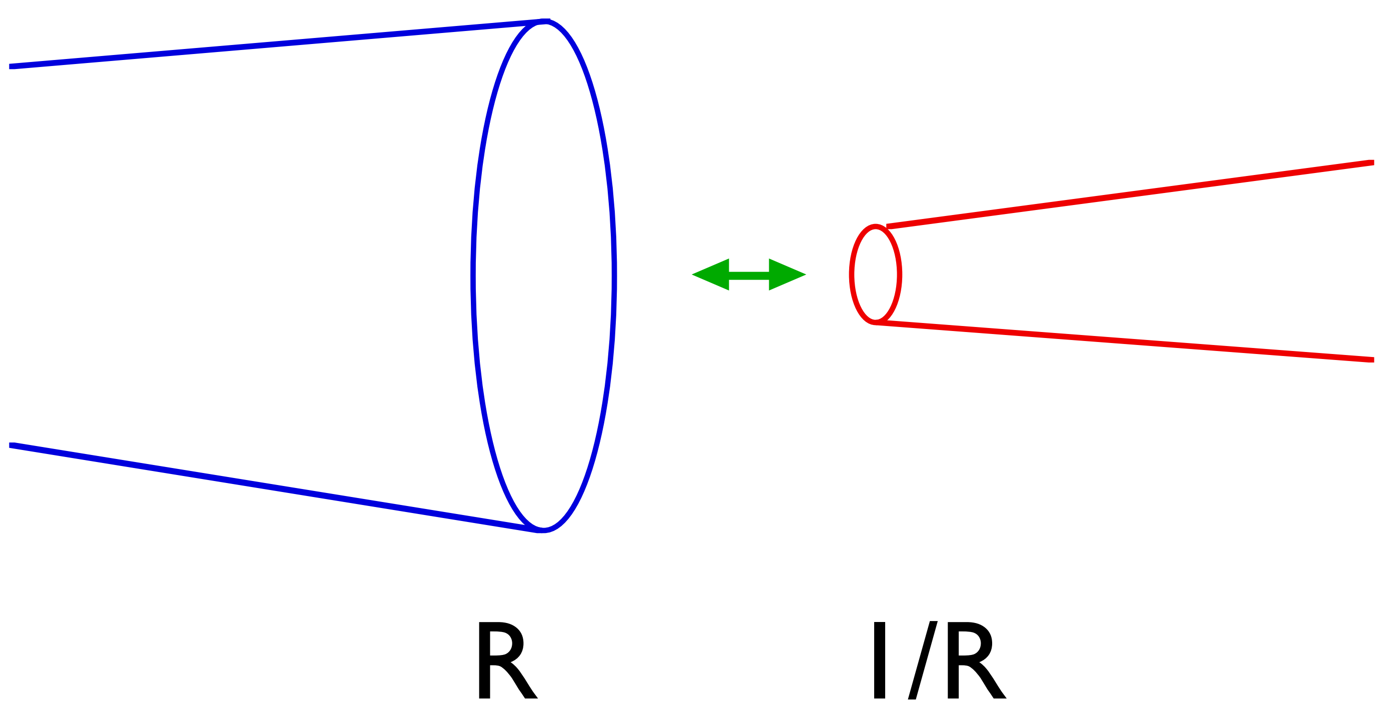
A Dabolkar and CH, 2002

Duality Symmetries

- Supergravities: continuous classical symmetry, broken in quantum theory, and by gauging
- String theory: discrete quantum duality symmetries; not field theory symms
- T-duality: perturbative symmetry on torus, mixes momentum modes and winding states
- U-duality: non-perturbative symmetry of type II on torus, mixes momentum modes and wrapped brane states
- Mirror Symmetry: perturbative symmetry on Calabi-Yau

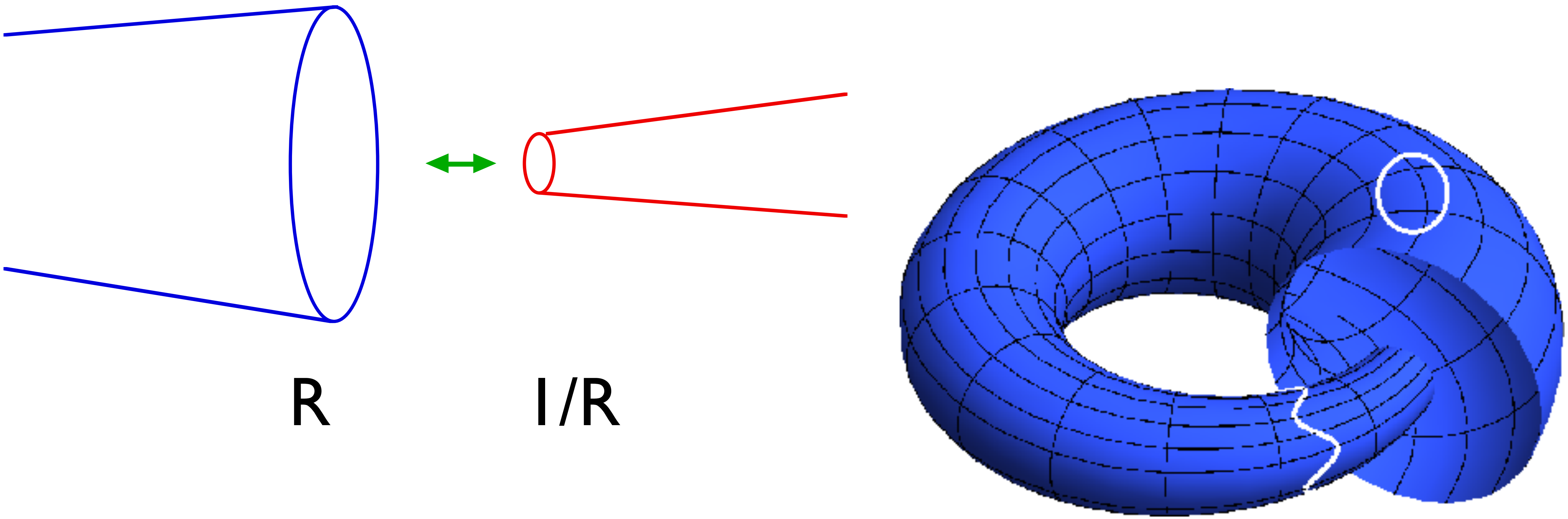
- Spacetime constructed from local patches
- All symmetries of physics used in patching
- Patching with diffeomorphisms, gives manifold
- Patching with gauge symmetries: bundles
- String theory has new symmetries, not present in field theory. New non-geometric string backgrounds
- Patching with T-duality: **T-FOLDS**
- Patching with U-duality: **U-FOLDS**
- Patching with MIRROR SYMM: **MIRROR-FOLDS**

T-fold patching



Glue big circle (R) to small (1/R)
Glue momentum modes to winding modes
(or linear combination of momentum and winding)
Not conventional smooth geometry

T-fold patching



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Non-Geometric Calabi-Yau Geometries

- Non-geometric reductions to D=4 Minkowski space
- For type II, N=2 SUSY in D=4. Fixes many moduli
- Mirrorfold — Mirror symmetry transitions
- Gauged D=4 sugras with N=2 Minkowski vacua
- At minimum of potential: SCFT — asymmetric Gepner model

- Suggestive of novel kind of doubling?
- New class of “compactifications”
- Bigger landscape?
- Could provide ways of escaping no-go theorems

- Kawai & Sugawara: Non-susy mirrorfolds
- Blumenhagen,Fuchs & Plauschinn. Gepner models from non-geometric quotient of CY CFT. Fixed point, so intrinsically stringy
- HIS: Gepner from asymm quotient of K3xT^2 CFT. Freely acting, so susy breaking scale not fixed at string scale. Sugra: good low energy description
- Non-geom from Stringy Scherk-Schwarz
  CH & Reid-Edwards, Reid-Edwards and Spanjaard
- G-theory Candelas Constantin Damian Larfors Morales K3 bundle over CP^1, U-duality monodromies.

Scherk-Schwarz reduction of Supergravity

- Supergravity in D dims:

Global duality G Scalars: G/H

Field $\phi \to g\phi$ $g \in G$

- <u>Reduce on S1</u>

$$\phi(x^m, y) = g(y)\varphi(x^m)$$

- Monodromy M on S1

$$\phi(x^m, 2\pi) = M\phi(x^m, 0) \qquad M = g(2\pi)g(0)^{-1}$$

e.g. $g(y) = \exp(yN)$ $M = \exp(2\pi N)$

Scherk-Schwarz reduction of Supergravity

- Reduce on T^n

Monodromy for each S^1 $\quad M_i \in G \qquad [M_i, M_j] = 0$

Conjugating gives equivalent theory

$$M_i' = g M_i g^{-1} \qquad g \in G$$

Consistent truncation of sugra to give gauged sugra in D-n dims.

Fields that are twisted typically become massive

Lifting to string theory

- Duality G broken to duality G($\mathbb{Z}$) CH&Townsend

G($\mathbb{Z}$) is automorphism group of charge lattice

Moduli space G($\mathbb{Z}$)\ G/H

- Monodromies must be in G($\mathbb{Z}$) CH '98

- Compatification with duality twists AD&CH '02

G($\mathbb{Z}$) conjugacy classes Masses quantized

$$M = \exp(2\pi N) \in G(\mathbb{Z})$$

- If D-dim theory comes from 10 or 11 dimensions by compactification on N (e.g. torus or K3), this lifts to "bundle" of N over T^n with G($\mathbb{Z}$) transitions

Torus Reductions with Duality Twists

If $N=T^d$ then have T^d "bundle" over T^n

For bosonic string $G(\mathbb{Z})=O(d,d;\mathbb{Z})$

Monodromies in T-duality group: T-fold

String theory on T^d: natural formulation on doubled torus T^{2d} with $O(d,d;\mathbb{Z})$ acting as diffeomorphisms

T-fold: T^{2d} bundle over T^n CH

Fully doubled: T^{2d} bundle over T^{2n} CH+ Reid-Edwards
Monodromies on doubled torus

K3 Sugra Reductions

IIA on K3: G=O(4,20), H=O(4)xO(20)

(2,2) Supergravity in d=6

IIA on K3xT^2: G=O(6,22), H=O(6)xO(22)

N=4 Supergravity in d=4

Scherk-Schwarz reduction: d=6 theory reduced on T^2 with monodromies $M_1, M_2 \in O(4,20)$

Gives gauged N=4 supergravity in d=4

Reid-Edwards and Spanjaard

Supersymmetry

Fermions: monodromies in Pin(4)xO(20)

Gravitini in **(2,1,1)x(1,2,1)** of SU(2)xSU(2)xO(20)

Preserving 16 SUSYs: $M_i \in O(20)$
Preserving 8 SUSYs: $M_i \in SU(2) \times O(20)$
Breaking all SUSY: $M_i \in SU(2) \times SU(2) \times O(20)$

K3

Compact Ricci flat Kahler 4-manifolds: K3 and T^4

K3 has SU(2) holonomy, hyperkahler. Manifold unique up to diffeomorphism.

Ricci flat metric depends on 22 moduli.

Moduli space

$$\mathcal{M} \cong O(3,19;\mathbb{Z})\backslash O(3,19)/O(3)\times O(19)$$

$O(3,19;\mathbb{Z})$: large diffeomorphisms of K3

K3 cohomology

$H^2(K3) = \mathbb{R}^{22}$ $H^0(K3) = H^4(K3) = \mathbb{R}$

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Metric on forms: $\qquad (\alpha_p, \beta_{4-p}) = \int \alpha_p \wedge \beta_{4-p}$

$H^2(K3) = \mathbb{R}^{3,19}$ $\qquad$ $H^0(K3) + H^4(K3) = \mathbb{R}^{1,1}$

$\mathbb{R}^{3,0}$ Self-dual harmonic 2-forms; hyperkahler structure

$\mathbb{R}^{0,19}$ Anti-self-dual harmonic 2-forms

$$H^*(K3) = \mathbb{R}^{4,20}$$

$$H^*(K3) = H^0(K3) + H^2(K3) + H^4(K3)$$

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Lattice of integral cohomology

$$\Gamma_{3,19} = H^2(K3;\mathbb{Z}) \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U$$

Preserved by $O(\Gamma_{3,19}) \sim O(3,19;\mathbb{Z})$

IIA String on K3

G=O(4,20;$\mathbb{Z}$) Automorphism group of CFT,
preserves charge lattice

$$\Gamma_{4,20} = H^*(K3;\mathbb{Z}) \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \oplus U$$

U: 2-dim lattice of signature (1,1)

$$\mathcal{M}_\Sigma \cong O(\Gamma_{4,20})\backslash O(4,20)/O(4) \times O(20)$$

O(3,19;$\mathbb{Z}$): large diffeomorphisms of K3

$\mathbb{Z}^{3,19}$: B-shifts

Rest of O(4,20;$\mathbb{Z}$) non-geometric

Compactify on T^2, monodromies $M_1, M_2 \in O(\Gamma_{4,20})$

Heterotic String Dual

IIA string on K3 ⟷ Heterotic string on T^4

CH&Townsend

IIA string on K3 "bundle" over T^2

⟷

Heterotic string on T^4 "bundle" over T^2

Monodromies in heterotic T-duality group $O(4,20;\mathbb{Z})$:

T-fold

Doubled picture: $T^{4,20}$ bundle over T^2

Compactification of String Theory with Duality Twists

AD&CH '02

Monodromies $M_i \in G(\mathbb{Z})$

Points in moduli space that give Minkowski-space minima of Scherk-Schwarz scalar potential

$\longleftrightarrow$

Points in moduli space fixed under action of $M_i \in G(\mathbb{Z})$

$M_i \in G(\mathbb{Z})$ has fixed point

$\longleftrightarrow$

$M_i \in G(\mathbb{Z})$ in elliptic conjugacy class

$G = SL(2, \mathbb{R})$

SL(2,$\mathbb{Z}$) Elliptic conjugacy classes
of order 2,3,4,6

$$M_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad M_3 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, \quad M_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad M_6 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbb{Z}_2, \mathbb{Z}_3 . \mathbb{Z}_4 . \mathbb{Z}_6$$

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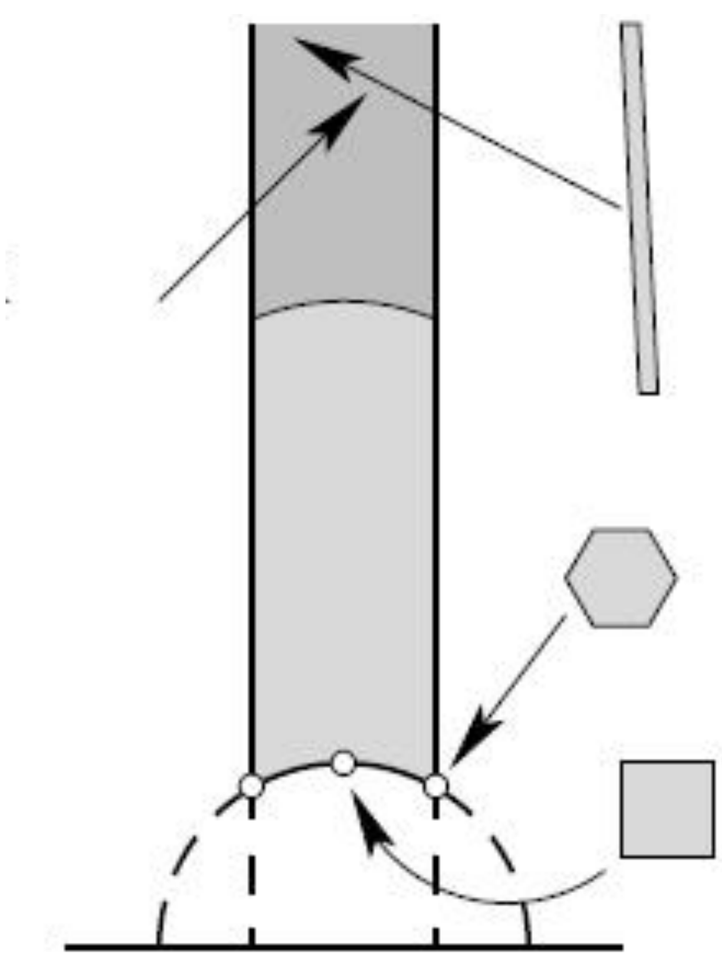

Corresponding fixed points in

$$SL(2,\mathbb{Z})\backslash SL(2,\mathbb{R})/U(1)$$

Minkowski Vacua and Orbifolds

AD&CH '02

At fixed point M_i generates $\mathbb{Z}^{p_i}$ $(M_i)^{p_i} = 1$

At this point in moduli space, construction becomes an orbifold, quotient by $M_i \times s_i$
G($\mathbb{Z}$) transformation together with shift in i'th S^1

$$s_i : y_i \to y_i + 2\pi/p_i$$

Geometric monodromies: orbifolds
T-duality monodromies: asymmetric orbifolds
K3 SCFT automorphisms: (asymmetric) Gepner models

Israel & Thiery

- Reduction with duality twist becomes orbifold at minima of potential, with explicit SCFT construction
- Reduction with duality twist gives extension of orbifold construction to whole of moduli space, identifies effective supergravity theory
- General point in moduli space not critical point. No Minkowski solution there but often e.g. domain wall solutions

String Constructions with Minkowski Vacua with N=2 SUSY

Need monodromies in elliptic conjugacy classes of O(20,4;$\mathbb{Z}$): i.e. in

$$M_i \in [O(4) \times O(20)] \cap O(4,20;\mathbb{Z})$$

SUSY $$M_i \in [SU(2) \times O(20)] \cap O(4,20;\mathbb{Z})$$

Any such monodromies will give Minkowski vacuum with N=2 SUSY

But finding such conjugacy classes is very hard open problem

Algebraic geometry constructs solutions

CY Mirror Symmetry

Moduli space of CY factorises

$$\mathcal{M}_{complex\ structure} \times \mathcal{M}_{Kahler}$$

Mirror CY has moduli spaces interchanged

$$\bar{\mathcal{M}}_{complex\ structure} = \mathcal{M}_{Kahler}$$

$$\bar{\mathcal{M}}_{Kahler} = \mathcal{M}_{complex\ structure}$$

K3 Mirror Symmetry

Moduli space doesn't factorise

$$\frac{O(4,20)}{O(4)\times O(20)}$$

No mirror symmetry: all K3's diffeomorphic

For <u>algebraic</u> K3, moduli space of CFTs factorises

$$\mathcal{M}_{complex}\times\mathcal{M}_{Kahler}=\frac{O(2,20-\rho)}{O(2)\times O(20-\rho)}\times\frac{O(2,\rho)}{O(2)\times O(\rho)}$$

Picard number ρ

Mirror symmetry interchanges factors

Mirrored Automorphisms

$$\hat{\sigma}_p := \mu^{-1} \circ \sigma_p^T \circ \mu \circ \sigma_p$$ CH, Israel and Sarti

$\mu : X \to \tilde{X}$ Mirror map for algebraic K3

σ_p Diffeomorphism of X $(\sigma_p)^p = 1$

σ_p^T Diffeomorphism of $\tilde{X}$ $(\sigma_p^T)^p = 1$

For suitable X, σ_p this acts on charge lattice by an O(4,20;Z) transformation that is elliptic and SUSY

Use such automorphisms for monodromies

Non-Geometric CY Vacua

- Minkowski vacuum with N=2 SUSY
- Asymmetric Gepner model of Israel & Thiery
- Explicit SCFT with Landau-Ginsurg formulation, asymmetric orbifold with discrete torsion
- D=4 gauged N=4 SUGRA, breaking to N=2. Outside classification of Horst,Louis,Smyth
- Massless sector: N=2 SUSY, STU model, or STU plus small number of hypermultiplets

Conclusions

- Non-geometries giving supersymmetric Minkowski vacua of string theory with few massless moduli

- Further non-geometries? Landscape? Physics?

- Mirrored automorphism involves K3 and its mirror. Some bigger picture? e.g. $X \times \tilde{X}$

- General mathematical structure? Generalised CY?