On marginal deformations of SCFT's in $D = 3$ and their AdS_4 duals

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Supersymmetric Theories, Dualities and Deformations
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in memory of Yassen STANEV

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Born the Fourth of July ... 1962

A great physicist, a wonderful colleague, a tender husband and father ... we will miss you a lot, thanks for what you left us with

Plan of the Talk

- Moduli problem in string theory and AdS_4/CFT_3
- Brane setup and $\mathcal{N} = 4$ quiver theories
- Super-gravity, holography and *petite Bouffe*
- Symmetries, spectrum, shortening
- $\mathcal{N} = 2$ preserving marginal deformations
- Conclusions

Moduli problem and AdS/CFT

Long-standing issue in String Theory
Fluxes generate (super)potentials that can help stabilisation in AdS then uplift ...
Some moduli deformations escape (gauged) supergravity description *e.g.* TsT deformation [Lunin, Maldacena; Imeroni; ...] for backgrounds with two commuting isometries

$$\tau \to \tau' = \frac{\tau}{1 + \gamma\tau}$$

γ real deformation parameter, τ modulus of *e.g.* $T^2 \in S^2_L \times S^2_R$
TsT breaks can break part or all super-symmetries
e.g. β deformation in $\mathcal{N} = 4$ SYM [.... Rossi, Sokatchev, STANEV] or other 'toric' SCFT's in $D = 4$

(Super)conformal manifold and *petite bouffe*

For $\mathcal{N}=1$ SCFT's in $D=4$ and for $\mathcal{N}=2$ SCFT's in $D=3$, super-conformal manifold $\mathcal{M}_{sc}$ Kähler quotient

$$\mathcal{M}_{sc} = \{\mathcal{W}^{CPO}_{\Delta=D-1=R}\}/G^{\mathbb{C}} \ .$$

$G^{\mathbb{C}}$ complexified global 'flavour' (non R-symmetry) group G
E.g. $\mathcal{N}=4$ SYM in $\mathcal{N}=1$ notation $U(1)_R$, $G=SU(3)$

$$dim_{\mathbb{C}}\mathcal{M}_c = 2 = 10-8$$

[Leigh, Strassler; Aharony, Kol,Yankielowicz; Green, Komargodski, Seiberg, Tachikawa, Wecht; ...]

$$W^{IJK}_{\mathbf{10}} = Tr(\Phi^I\{\Phi^J,\Phi^K\})$$

Holographic description, (supersymmetric) Higgs/Stückelberg mechanism: *petite bouffe* $\{V,\varphi\}_{m=0} \to \mathcal{V}_{m\neq 0}$

$$\partial_\mu \mathcal{J}^\mu_{\Delta=D-1} = 0 \, , \ \mathcal{L}_{\Delta=D} \quad \to \quad \partial_\mu \mathcal{J}^\mu_{\Delta=D-1+\gamma} = \mathcal{L}_{\Delta=D+\gamma}$$

Generalization to higher spins: *La Grande Bouffe* [MB, Morales, Samleben; Beisert; ...]

$$\partial \mathcal{J}^{(s)}_{\Delta=s+D-2} = 0 \, , \ \mathcal{L}^{(s-1)}_{\Delta=s+D-1} = 0 \quad \to \quad \partial \mathcal{J}^{(s)}_{\Delta=s+D-2+\gamma} = \mathcal{L}^{(s-1)}_{\Delta=s+D-1+\gamma}$$

Brane setup for $\mathcal{N} = 4$ SCFT's in $D = 3$

Brane creation-annihilation [Hanany, Witten], Boundary States [MB, Stanev; ...]

Brane	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$D3$	—	—	—	|—|	.	.	.	.	.	.
$D5$	—	—	—	.	.	.	.	—	—	—
$NS5$	—	—	—	.	—	—	—	.	.	.

$\mathcal{N} = 4$ in $D = 3$: 8 $\mathcal{Q}^{a\dot{a}}_{\alpha}$, $Osp(4|4) \supset SO(2,3) \times SO(4)$
R-symmetry $SO(4) = SO(3)^{456}_{L} \times SO(3)^{789}_{R} = SU(2)_V \times SU(2)_H$
Hyperkähler Moduli space $\mathcal{M} = \mathcal{M}_v \times \mathcal{M}_h$

- Coulomb branch: $\mathcal{M}_v$ receives quantum corrections
- Higgs branch: $\mathcal{M}_h$ NO corrections

No $\mathcal{N} = 4$ preserving exactly marginal deformations
NO $\mathcal{N} = 4$ Higgsing / petite bouffe [De Alwis, Louis, Mc Allistair, Triendl; ...]
... neither $\mathcal{N} = 3$ preserving ('quantised')
... yet there may be $\mathcal{N} = 2$ preserving

Quiver Theories

$\mathcal{N} = 4$ gauge theories: vector-plets and hypermultiplets,

- Electric quiver: D3-branes suspended between NS5-branes and intersecting D5-branes.
- Magnetic quiver: roles of NS5 and D5 exchanged

Brane data $\{N_a, \ell_a\}$ and $\{\hat{N}_{\hat{a}}, \hat{\ell}_{\hat{a}}\}$, linking numbers

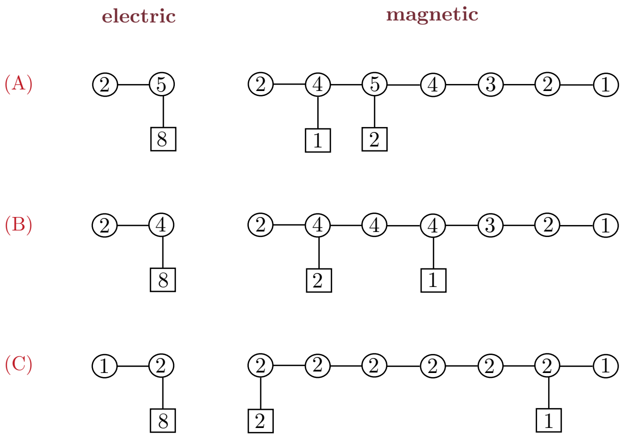
$$\sum_a N_a = K \; (D5{-}branes) \quad , \quad \sum_{\hat{a}} \hat{N}_{\hat{a}} = \hat{K} \; (NS5{-}branes)$$

Electric quiver: $\hat{K} - 1$ 'gauge' nodes, $[g_{YM}] = [M] \sim |\Delta x_3|/\alpha'$
At IR fixed point, coexisting global flavor symmetries of SCFT

- D5-branes $\prod_a U(N_a)$ manifest in 'electric quiver',
- NS5-brane $\prod_{\hat{a}} U(\hat{N}_{\hat{a}})$ manifest in mirror 'magnetic quiver'.

Balanced nodes ...

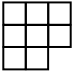
A, B, C of Linear Quivers



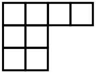
Electric and magnetic quivers of theories (A, B, C) with $N = 8$. In the IR, theories (A, B, C) have same $SU(8) \times SU(2) \times U(1)$ flavor symmetry but different operator content.

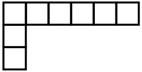
'Fine-prints' and Young tableaux

Linking numbers, conservation of D3-brane charge
D5's ℓ_a (from right to left), NS5's $\hat{\ell}_{\hat{a}}$ (from left to right)
partitions of $N := \sum_{\hat{a}} \hat{N}_{\hat{a}} \hat{\ell}_{\hat{a}} = \sum_a N_a |\ell_a| \leftrightarrow$ Young tableaux ρ, $\hat{\rho}$
ρ: N_a rows of $|\ell_a|$ boxes, $\hat{\rho}$: $\hat{N}_{\hat{a}}$ rows of $\hat{\ell}_{\hat{a}}$ boxes
Partial ordering $\rho^T > \hat{\rho}$: non trivial Higgs branch, 'good' [Gaiotto, Witten]

$\hat{\rho}_A =$ 

$\rho =$ 

$\hat{\rho}_B =$ 

$\hat{\rho}_C =$ 

Supergravity description

AdS_4 compactifications of Type IIB with $\mathcal{N} = 4$ gauged SUGRA

$$AdS_4 \times S_L^2 \times S_R^2 \times_w \Sigma$$

Σ, open Riemann surface: disk (linear quiver) or annulus (circular quiver) [D'Hoker, Estes, Gutperle; Assel, Bachas, ...]

Super-conformal symmetry $\mathfrak{osp}(4|4) \supset SO(4)$ isometry of $S_L^2 \times S_R^2$

Two harmonic functions $h_{1,2}(z, \bar{z})$ positive in interior of Σ, vanish at points on the boundary

Henceforth Σ infinite strip $0 \leq \mathrm{Im} z \leq \pi/2$ (disk, linear quiver)

Singularities:

- N_a D5-branes at $\mathrm{Re}z = \delta_a$, $a = 1, \cdots, p$, on upper boundary,
- $\hat{N}_{\hat{a}}$ NS5-branes at $\mathrm{Re}z = \hat{\delta}_{\hat{a}}$, $\hat{a} = 1, \cdots, \hat{p}$, on lower boundary

Quantization conditions

$$\pi \ell_a = -2 \sum_{\hat{a}=1}^{\hat{p}} \hat{N}_{\hat{a}} \arctan(e^{-\delta_a + \hat{\delta}_{\hat{a}}}) , \quad \pi \hat{\ell}_{\hat{a}} = 2 \sum_{a=1}^{p} N_a \arctan(e^{-\delta_a + \hat{\delta}_{\hat{a}}})$$

... seem to fix all moduli parameters ...

Spectrum and $\mathcal{N} = 4$ multiplets

Barring excited-string modes, single-particle states either from 10d graviton multiplet or from lowest-lying modes of open strings living on penta-branes. Both have $S_{\rm Max} \leq 2$.
Organized in three series of representations of

$$\mathfrak{osp}(4|4) \supset SO(4) = SU(2)_L \times SU(2)_R$$

- '1/2 BPS' $B_1[0]_L^{(L,0)}$ and $B_1[0]_R^{(0,R)}$ series with $S_{\rm Max} \leq 1$
- '1/4 BPS' $B_1[0]_{L+R}^{(L,R)}$ series $(LR \neq 0)$ with $S_{\rm Max} \leq 3/2$
- 'semi-short' $A_2[0]_{L+R+1}^{(L,R)}$ series with $S_{\rm Max} \leq 2$

Legenda: $HWS = [S]_\Delta^{(L,R)}$, $B_1[0]_L^{(L,0)} \sim H^{2L}$, $B_1[0]_R^{(0,R)} \sim \tilde{H}^{2R}$
Ultrashort 'singleton' representations, free (twisted) hypers

$$H^a = \varphi^a + \theta_\alpha^{a\dot{a}} \zeta_{\dot{a}}^\alpha \quad \sim \quad B_1[0]_{1/2}^{(1/2;0)} = [0]_{1/2}^{(1/2;0)} \oplus [1/2]_1^{(0;1/2)}$$

$$\tilde{H}^{\dot{a}} = \tilde{\varphi}^{\dot{a}} + \theta_\alpha^{a\dot{a}} \tilde{\zeta}_a^\alpha \quad \sim \quad B_1[0]_{1/2}^{(0;1/2)} = [0]_{1/2}^{(0;1/2)} \oplus [1/2]_1^{(1/2;0)}$$

$\mathcal{N} = 4$ **Multiplet**	**String mode**	**gauged SUGRA**
$A_2[0]_1^{(0;0)}$	Graviton	YES
$B_1[0]_1^{(1;0)}$	D5 gauge bosons	YES
$B_1[0]_1^{(0;1)}$	NS5 gauge bosons	YES
$B_1[0]_L^{(L>1;0)}$	Closed strings $L \in \mathbb{N}$	only $L = 2$
$B_1[0]_L^{(L>1;0)}$	Open F-strings $L \in \frac{1}{2}\lvert\ell_a - \ell_b\rvert + \mathbb{N}$	only $L = 2$
$B_1[0]_R^{(0;R>1)}$	Closed strings $R \in \mathbb{N}$	only $R = 2$
$B_1[0]_R^{(0;R>1)}$	Open D-strings $R \in \frac{1}{2}\lvert\hat{\ell}_{\hat{a}} - \hat{\ell}_{\hat{b}}\rvert + \mathbb{N}$	only $R = 2$
$B_1[0]_{L+R}^{(L\geq 1;R\geq 1)}$	Kaluza Klein gravitini $(L,R\in\mathbb{N})$	NO
$A_2[0]_{1+L+R}^{(L>0;R>0)}$	Kaluza Klein gravitons $(L,R\in\mathbb{N})$	NO
$A_1[S > 0]_{1+S+L+R}^{(L;R)}$	Stringy excitations	NO

Shortening and re-combination

Short $\mathcal{N}=4$ multiplets see e.g. [Dolan; Cordova, Dumitrescu, Intriligator; ...] ... with some care

$$A_1[S]^{(L,R)}_{1+S+L+R}\,(S>0) \quad , \quad A_2[0]^{(L,R)}_{1+L+R}\,(S=0) \quad , \quad B_1[0]^{(L,R)}_{L+R}$$

Stress-energy tensor $\leftrightarrow$ graviton multiplet 'semishort'

$$A_2[0]^{(0;0)}_1 = [0]^{(0;0)}_1 \oplus [0]^{(0;0)}_2 \oplus [1]^{(1;0)}_2 \oplus [1]^{(0;1)}_2 \oplus [2]^{(0;0)}_3 \oplus \text{fermions}$$

'Electric' flavor current $\leftrightarrow$ L-vector multiplet '1/2 BPS'

$$B_1[0]^{(1;0)}_1 = [0]^{(1;0)}_1 \oplus [1]^{(0;0)}_2 \oplus [0]^{(0;1)}_2 \oplus \text{fermions}$$

'Magnetic' flavor current $\leftrightarrow$ R-vector multiplet '1/2 BPS'

$$B_1[0]^{(0;1)}_1 = [0]^{(0;1)}_1 \oplus [1]^{(0;0)}_2 \oplus [0]^{(1;0)}_2 \oplus \text{fermions}$$

At unitarity threshold can re-combine *e.g.*

$$L[0]^{(0;0)}_1 = A_2[0]^{(0;0)}_1 \oplus B_1[0]^{(1;1)}_2$$

and get 'mass' / 'anomalous dimension' (*petite* bouffe, $S_{\rm Max}=2$)

NO $\mathcal{N} = 4, 3$ preserving Marginal Deformations

Scalar operators of dimension $\Delta = 3$ in $\mathcal{N} = 4$ multiplets:

- NO top components
- NO dead-end components

Yet, relevant $\mathcal{N} = 4$ deformations:

- Scalar $[0]_2^{(0;0)}$ in stress-tensor multiplet can trigger a universal $\mathcal{N} = 4$ mass deformation
- Scalars in electric and magnetic flavor-current multiplets $B_1[0]_1^{(1;0)}$ and $B_1[0]_1^{(0;1)}$: triplets of flavor masses and Fayet-Iliopoulos terms

$\mathcal{N} = 3$ preserving 'deformations' $W = kTr(\Phi^2)$ (quantized)

Looking for $\mathcal{N} = 2$ preserving Marginal Deformations

Basic $\mathcal{N} = 2$ multiplets ($HWS = [S]^{(r)}_{\Delta}$)
conserved stress-tensor multiplet

$$A_1\bar{A}_1[1]^{(0)}_2 = [1]^{(0)}_2 \oplus [\tfrac{3}{2}]^{(\pm 1)}_{5/2} \oplus [2]^{(0)}_3 \ ,$$

vector current multiplet

$$A_2\bar{A}_2[0]^{(0)}_1 = [0]^{(0)}_1 \oplus [\tfrac{1}{2}]^{(\pm 1)}_{3/2} \oplus [0]^{(0)}_2 \oplus [1]^{(0)}_2 \ ,$$

Chiral multiplets ($r > 0$)

$$L\bar{B}_1[0]^{(r)}_r = [0]^{(r)}_r \oplus [\tfrac{1}{2}]^{(r-1)}_{r+\frac{1}{2}} \oplus [0]^{(r-2)}_{r+1} \ .$$

Anti-chiral multiplets ($r < 0$)

$$B_1\bar{L}[0]^{(r)}_{|r|} = [0]^{(r)}_{|r|} \oplus [\tfrac{1}{2}]^{(r+1)}_{|r|+\frac{1}{2}} \oplus [0]^{(r+2)}_{|r|+1} \ .$$

$\mathcal{N} = 2$ Marginal Deformations

'Superpotential' multiplet $L\bar{B}_1[0]_2^{(2)}$ (and its conjugate $B_1\bar{L}[0]_2^{(-2)}$)

$$L\bar{B}_1[0]_2^{(2)} = [0]_2^{(2)} \oplus [\tfrac{1}{2}]_{5/2}^{(1)} \oplus [0]_3^{(0)}$$

Can be lifted only by recombination with a vector multiplet

$$L\bar{B}_1[0]_2^{(2)} \oplus B_1\bar{L}[0]_2^{(-2)} \oplus A_2\bar{A}_2[0]_1^{(0)} \ \rightarrow \ L\bar{L}[0]_1^{(0)} \ .$$

Super-symmetric Higgsing / *'petite' bouffe* $S_{\mathrm{Max}} = 1$

$$\partial_\mu \mathcal{J}^\mu = 0 \, , \ \mathcal{L} \quad \rightarrow \quad \partial_\mu \mathcal{J}^\mu = \mathcal{L} \ .$$

Superconformal manifold $\mathcal{M}_{sc}$ Kähler quotient

$$\mathcal{M}_{sc} = \{\lambda_i | D^a = 0\}/G = \{\lambda_i\}/G^{\mathbb{C}} \ .$$

D^a Kähler moment-maps, a adjoint index of global G

Moral:

look for $\mathcal{N} = 2$ 'superpotential' inside $\mathcal{N} = 4$ supermultiplets

$\mathfrak{osp}(4|4) \supset \mathfrak{osp}(2|4) \oplus \mathfrak{u}(1)_F$ decomposition

$\mathfrak{u}(1)_F$ 'accidental' flavor symmetry $\perp$ R-symmetry $\mathfrak{u}(1)_R$: $r = L+R$

Potential $\mathcal{N}=4$ representations with $\Delta = 2$ scalars:

$$B_1[0]^{(L,R)}_{L+R} \;\; \textit{with} \;\; L+R = 1,2 \quad \textit{or} \quad A_2[0]^{(L,R)}_{1+L+R} \;\; \textit{with} \;\; L+R = 0,1$$

Lowest entries: stress-tensor and vector-current ... NO GOOD

Good candidates with marginal superpotential $L\bar{B}_1[0]^{(2)}_2$ (in box)

- From open or closed strings (within gauged-supergravity)

$$B_1[0]^{(2;0)}_2 \; = \; L\bar{L}[0]^{(0)(0)}_2 \oplus \left[L\bar{A}_2[0]^{(1)(1)}_2 \;\oplus\; \boxed{L\bar{B}_1[0]^{(2)(2)}_2} \oplus c.c.\right],$$

- from Kaluza-Klein gravitini

$$B_1[0]^{(1;1)}_2 \; = \; L\bar{L}[0]^{(0)(0)}_2 \oplus \Big[L\bar{L}[0]^{(0)(2)}_2 \oplus L\bar{A}_2[0]^{(1)(1)}_2 \oplus L\bar{A}_2[0]^{(1)(-1)}_2 \oplus$$

$$\boxed{L\bar{B}_1[0]^{(2)(0)}_2} \oplus c.c.\Big] \oplus \left[L\bar{L}[\tfrac{1}{2}]^{(0)(1)}_{5/2} \oplus L\bar{A}_1[\tfrac{1}{2}]^{(1)(0)}_{5/2} \oplus c.c. \right] \oplus L\bar{L}[0]_3^{(}$$

- From exotic multi-particle states (violate isospin rule)

$$B_1[0]^{(3/2;1/2)}_2 \; = \; \left[L\bar{L}[0]^{(0)(1)}_2 \oplus L\bar{A}_2[0]^{(1)(2)}_2 \oplus L\bar{A}_2[0]^{(1)(0)}_2 \;\oplus\; \boxed{L\bar{B}_1[0]^{(2)(1)}_2} \oplus c.c.\right]$$

$$\oplus \; L\bar{L}[\tfrac{1}{2}]^{(0)(0)}_{5/2} \oplus \left[L\bar{A}_2[0]^{(2)(1)}_3 \oplus c.c. \right].$$

$\mathcal{N}=4$ quivers in $\mathcal{N}=2$ language

$\mathcal{N}=2$ sub-algebra:

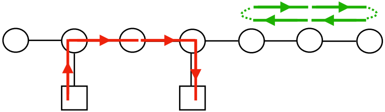
- vector-plets decompose into (V,Φ) in Adj representation
- hyper-multiplets into pairs $(q,\tilde{q})$ in conjugate representations

$\mathcal{N}=2$ superpotential: $W=\sum_\ell q_{\ell-1,\ell}\Phi_\ell\tilde{q}_{\ell,\ell-1}$
Recall: D5-branes grouped in stacks by their linking number, indicating circular (gauge) node to which they attach.
Quiver data specified by two sets of $\hat{K}-1$ non-negative integers:

- flavor $\mathbf{N}=\{N_\ell\}$ (magnetic quiver A: $\mathbf{N}=\{0,1,2,0,0,0,0\}$)
- gauge $\mathbf{n}=\{n_\ell\}$ (magnetic quiver A: $\mathbf{n}=\{2,4,5,4,3,2,1\}$)

Chiral operators H^{2L} on Higgs branch, in $B_1[0]_L^{(L;0)}$ of $\mathcal{N}=4$ SCA, singlets of $SU(2)_{C/R}$ with $\Delta=L$ and $SU(2)_{H/L}$ isospin $L=r$.
Absolutely protected, survive infrared SCFT.

Chiral Operators on the Higgs branch

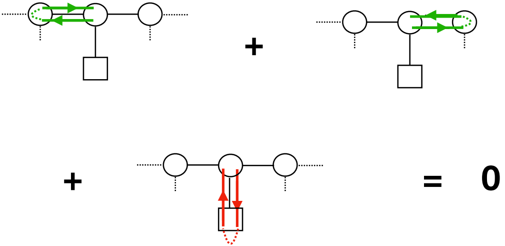


Two chiral operators on Higgs branch of magnetic quiver B. Open-string operator (in red) in bi-fundamental of flavor group $U(2) \times U(1)$, while closed-string operator (in green) flavor singlet. Both represent marginal superpotential deformations since they have length 4, and hence belong to $B_1[0]_2^{(2;0)}$ multiplets.

F-flatness condition on the Higgs branch

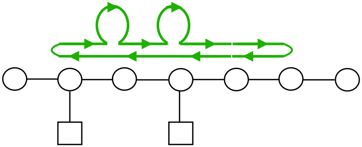
F-term conditions for $q, \tilde{q}$ on the Higgs branch

$$\tilde{q}_{\ell,\ell+1}\, q_{\ell+1,\ell} + q_{\ell,\ell-1}\, \tilde{q}_{\ell-1,\ell} + \tilde{q}_{\ell,f_\ell}\, q_{f_\ell,\ell} = 0$$



Graphical representation of the F-flatness conditions on the Higgs branch, as linear relations among cut-open string segments.
The dotted red/green semicircles stand for summation over free flavor/gauge indices of the open strings.

Chiral operators on the Mixed Branch



Gauge-invariant products of chiral fields from both hyper multiplets (line segments) and vector multiplets (bubbles).
The closed string in figure has 8 line segments and 2 bubbles, and transforms in the representation $B_1[0]_6^{(4;2)}$.
F-term condition

$$q_{f_\ell,\ell}\,\Phi_\ell = \Phi_\ell\,\tilde{q}_{\ell,f_\ell} = 0 \quad q_{\ell+1,\ell}\,\Phi_\ell \sim \Phi_{\ell+1}\,q_{\ell+1,\ell} \quad \tilde{q}_{\ell,\ell+1}\,\Phi_{\ell+1} \sim \Phi_\ell\,\tilde{q}_{\ell,\ell+1}$$

Chiral Ring: summary for $\Delta \leq 2$

- $\mathbf{\Delta = 1}$ in conserved-current multiplets of $\mathcal{N} = 4$. Length-2 'open strings' in Adj of $U(N_\ell)$ flavor groups, Length-2 'closed string' per each of the $\hat{K} - 2$ 'internal' nodes, subject to $\hat{K} - 1$ F-term conditions, from $Tr(\Phi_\ell)$. Number of independent operators matches dimension of flavor group $\prod_\ell U(n_\ell)/U(1)$. Overall $U(1)$ acts trivially and decouples.
- $\mathbf{\Delta = 3/2}$: No length-3 'closed strings' in accordance with integer L, R 'isospin' selection rule for spin-0 closed strings. Length-3 'open strings' from neighbouring pairs of flavour (square) nodes in the bi-fundamental of $U(N_\ell) \times U(N_{\ell+1})$.
- $\mathbf{\Delta = 2}$: Length-4 chiral operators in $B_1[0]_2^{(2;0)}$ or $B_1[0]_2^{(0;2)}$: sought for marginal $\mathcal{N} = 2$ superpotential deformations. 'Open strings' in symmetric product of Adj of flavor group, or in bi-fundamental of $U(N_\ell) \times U(N_{\ell+2})$ (if any). Bound states of two open strings: "second adjoint" representation, adjoint representation, and 4th rank antisymmetric representation.

Chiral Ring and Holography

Emerging pattern: at level $\Delta = r$ single-string chiral operators, either closed strings or open strings in the bi-fundamental of $U(N_\ell) \times U(N_{\ell'})$, subject to

$$r = 1 + n \quad \textit{closed strings}, \qquad r = \frac{1}{2}|\ell - \ell'| + n \quad \textit{open strings}$$

Match precisely holographic dual Type IIB string spectrum

Also obtained as scaling dimensions of monopole operators on Coulomb branch of magnetic quiver, in agreement with mirror symmetry

For linear quivers, can choose basis where all chiral operators = multi-particle bound states of open strings *e.g.* built in terms of 'meson' matrices $M_i{}^j = \tilde{q}_i^u q_u^j$.

Using F-term condition on Higgs branch to "fold and slide" closed strings along 'internal' nodes until they hit the boundary and 'annihilate' into open strings.

Caveat: This does not work for circular quivers, with no boundary, that can support irreducible closed winding strings.

Moduli spaces

Nilpotent orbits, Slodowy slice, Kraft-Procesi transition ... please ask Ami, Santiago or directly Claudio]

$\overline{\mathcal{O}}_\rho$ closure of nilpotent orbit associated to partition ρ of N.

Orbit $\mathcal{O}_\rho$ consists of all $N \times N$ nilpotent matrices whose Jordan normal form has blocks of sizes given by the partition ρ. Closure includes orbits of all smaller partitions.

Slodowy slice $\mathcal{S}_\rho$ associated to ρ partition: transverse slice to orbit $\mathcal{O}_\rho$ in the space freely generated by adjoint-valued variables.

Higgs branch of electric theory = Coulomb branch of magnetic theory given by the intersection

$$\mathcal{H}_e = \mathcal{C}_m = \mathcal{S}_\rho \cap \overline{\mathcal{O}}_{\hat{\rho}^T} \ ,$$

Higgs branch $\mathcal{H}_m$ of magnetic theory = Coulomb branch $\mathcal{C}_e$ of electric theory given by 'mirror' intersection

$$\mathcal{H}_m = \mathcal{C}_e = \mathcal{S}_{\hat{\rho}} \cap \overline{\mathcal{O}}_{\rho^T} .$$

Moduli spaces for A,B,C

For ‘our’ models A, B, C with $N = 8$ and different partitions, 8×8 ‘meson’ matrix $M_i^j = \tilde{q}_i^{u_1} q_{u_1}^j$

$$\mathcal{H}_e^A = \mathcal{C}_m^A = \bar{\mathcal{O}}_{\hat{\rho}_A^t} = \{M_{8\times 8} : TrM = TrM^2 = 0, M^3 = 0, rk(M) \leq 5\}$$

$$\mathcal{H}_e^B = \mathcal{C}_m^B = \bar{\mathcal{O}}_{\hat{\rho}_B^t} = \{M_{8\times 8} : TrM = TrM^2 = 0, M^3 = 0, rk(M) \leq 4\}$$

$$\mathcal{H}_e^C = \mathcal{C}_m^C = \bar{\mathcal{O}}_{\hat{\rho}_C^t} = \{M_{8\times 8} : TrM = TrM^2 = 0, M^3 = 0, rk(M) \leq 2\}$$

where $rk(M) \leq n_3^{(\ell=1)}$ is the rank of the ‘meson’ matrix M.

Global symmetry organizes chiral operators

The global 'flavour' symmetry of $\mathcal{H}_e^{A,B,C}$ is $SU(8)$.
In magnetic description, A_7 formed by balanced nodes.
Chiral operator content up to $\Delta = r = 2$

$$\mathcal{Z}(\mu_i, t_1) = 1 + \mu_1\mu_7 t_1^2 + (\mu_1^2\mu_7^2 + \mu_2\mu_6 + \mu_1\mu_7)t_1^4 + \ldots$$

with μ_i, $i = 1, \ldots, 7$ $SU(8)$ fugacities, $t_{1(2)}$ $SU(2)_{H(C)}$ fugacities
Other branches: $U(1)\times U(2)/U(1) \simeq SU(2)\times U(1)$ global symmetry

$$\begin{aligned}
A &: \quad 1 + (\mu^2 + 1)t_2^2 + \mu(\alpha + \alpha^{-1})t_2^3 + (\mu^4 + \mu^2 + 1)t_2^4 \\
B &: \quad 1 + (\mu^2 + 1)t_2^2 + (\mu^4 + \mu^2 + 1)t_2^4 + \mu(\alpha + \alpha^{-1})t_2^4 \\
C &: \quad 1 + (\mu^2 + 1)t_2^2 + (\mu^4 + \mu^2 + 1)t_2^4
\end{aligned}$$

$\mathcal{N} = 2$ super-conformal manifold: all combinations of chiral operators with $\Delta = r = 2$, then quotient by global symmetry $SU(8) \times SU(2) \times U(1) \times U(1)_F$ (68 generators).
Even neglecting 'mixed' branches, get a formidable number!!!

Counting super-marginal deformations

Counting by means of plethystic techniques: Hilbert series, Molien-Weyl integrals, ... see e.g. [Benvenuti, Hanany; Feng, He; ...]
Relevant integral for (the most interesting) theory B

$$\oint \frac{dz}{z} \oint \frac{dw}{w} \int d\mu_{SU(8)} d\mu_{SU(2)} \mathcal{Z}$$

with $d\mu_G$ Haar measure, z (w) fugacity for $U(1)_F$ ($U(1)$ 'magnetic') and

$$\mathcal{Z} = \mathrm{PE}\{([2,0,0,0,0,0,2;0]+[0,1,0,0,0,1,0;0]+[1,0,0,0,0,0,1;0])z^2q^2 \\ +([\vec{0};4]+[\vec{0};2]+[\vec{0};1](w+w^{-1})+[\vec{0};0])z^{-2}q^2+[1,0,0,0,0,0,1;2]q^2\}$$

$[n_1,\ldots,n_7;n]$ denotes character of $SU(8)\times SU(2)$ irrep with given Dynkin labels and $\mathrm{PE}[f(t)] = \exp\sum_{n=1}^{\infty}[f(t^n)-f(0)]/n$

Conclusions and Outlook

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- Holographic description of $\mathcal{N} = 2$ super-conformal manifold $\mathcal{M}_{s-c} = \{\mathcal{W}_2^{(+2)}\}/G^{\mathbb{C}}$ and supersymmetric petite bouffe
- Embedding into $\mathcal{N} = 4$, identification of $\Delta = r = 2$ chiral operators, global symmetry *e.g.* $SU(8)\times SU(2)\times U(1)\times U(1)_F$ and counting with plethystic techniques
- Find holographic duals of 'deformed' quivers
- Generalize to circular quivers *i.e.* $\Sigma =$ annulus ...

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