Exact results in SUSY Spin(7) theories

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Motivations 1

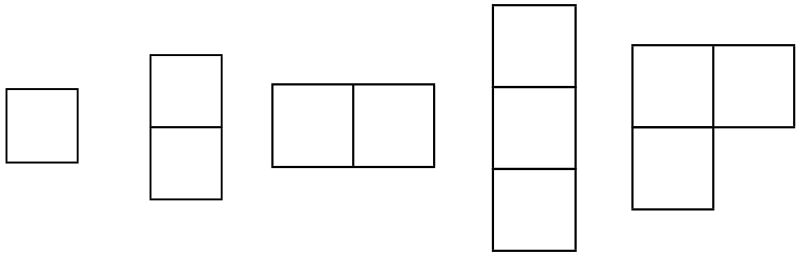
- asymptotically-free gauge theories (IR strong)

  → various (low-enrgy) phases

  confinement, Higgs, Coulomb and etc…

- Phases depend on:

  temperature, space-time dimensions, matter contents Nf (Number of flavors), representations of matters:



gauge groups: (S)U, (S)O, USp, Spin, Pin, Exceptionals…

Today

Motivations 2

- To study strongly-coupled or non-perturbative regime with power of SUSY (holomorphy) → analytically extract exact results!!
- Recent developments about 3d theories

3d & 4d physics are similar, but in some sense “3d” is more richer.

Chern-Simons terms, non-trivial U(1) dynamics and bosonization

Motivations 3

- Many 3d dualities were found now: Seiberg-like, CS-type, non-SUSY (bosonization)

- Check of the 3d dualities/ analysis of the low-energy physics → (mapping of) monopole operators (Coulomb branch operators)

- difficult ··· → power of supersymmetry

Today’s topic

- 3d N=2 SUSY **Spin(7)** theories and their low-energy dynamics, especially “confinement”
- First non-trivial case of Spin(N)
- **vector** & **spinorial** representations
- 3d/4d relation

Plan of the talk

1. Introduction to 3d N=2 SUSY gauge theories
2. Spin(7) analysis in 3d
3. Some checks of our analysis (3d/4d)

3d N=2 SUSY gauge theories

[Aharony-Hanany-Intriligator-Seiberg-Strassler ’97]

3d gauge theories

- 3d gauge coupling is super-renormalizable and relevant.

$$[g^2] = M \qquad \hat{g}^2 := \frac{g^2}{E} \xrightarrow[E \to 0]{} \infty$$

3d U(1) dynamics is also "IR strong" and non-trivial.

ex. 3d "compact" U(1) QED shows confinement and chiral symmetry breaking.

Along the Coulomb branch, we have U(1) gauge theories which are still non-trivial.

3d N=2 theories

[Aharony-Hanany-Intriligator-Seiberg-Strassler ’97]
[de Boer-Hori-Oz ’97]

- 3d UV Lagrangian = simple dimensional reduction of 4d N=1 SUSY

Chiral Superfield

$$\Phi(x,\theta) = \underset{\text{complex scalar}}{\phi} + \theta \underset{\text{fermion}}{\psi} + \theta^2 \overset{\text{auxiliary field}}{F} + \cdots$$

This gives us flat directions (Higgs branch of the moduli space).

Spin(7) cases

$S(x,\theta)$: 8 dimensional spinor rep. in Spin(7)

$Q(x,\theta)$: 7 dimensional vector rep. in Spin(7)

We need various gauge invariants.

$$M_{SS} := SS$$
$$M_{QQ} := QQ$$
$$\vdots$$

3d N=2 theories

[Aharony-Hanany-Intriligator-Seiberg-Strassler '97]
[de Boer-Hori-Oz '97]

Vector superfield

$$V(x,\theta,\bar{\theta}) = \theta\sigma^{\mu}\bar{\theta}A_{\mu} + \theta\sigma^{3}\bar{\theta}A_{3} + (\text{gaugino}) + \cdots \qquad \mu = 0,1,2$$

↑

adjoint scalars (Coulomb branch of moduli)

$G \xrightarrow{\langle A_3 \rangle \neq 0} U(1)^r$ → non-trivial U(1) dynamics

- 3d photon is dual to a scalar

$$\sigma := A_3$$

$$\partial_{\mu}a := \epsilon_{\mu\nu\rho}F^{\nu\rho}$$

two real scalars
= one complex scalar

$$V_i = \exp(\sigma_i + i a_i) \quad (i = 1, \cdots, r)$$

$\phi_{adj} = \phi^{i}H_{i}$

Classically, there are r=rank G CB coordinates.

Low-energy dynamics

UV Lagrangian

$$\mathcal{L} = \underbrace{\int d^4\theta \bar{\Phi} e^V \Phi}_{\text{Matter action}} + \underbrace{\int d^2\theta W_\alpha W^\alpha}_{\text{gauge action}} + \text{h.c.}$$

- (quantum) flat directions (massless modes) ?
- Non-perturbative effects ?
- Exact superpotential ? $\mathcal{L}_{eff} = \int d^2\theta \, W_{eff}(\Phi)$
- Correct Coulomb branch operators (monopole operators) ?

There are two branches of moduli space: Higgs (meson, baryon) & Coulomb

Their interactions, low-energy superpotential for these massless modes?

Classical vs Quantum pictures?

Monopoles

- Along the Coulomb branch, we have a compact U(1).
- Theory admits monopoles (in 3d, these are instantons)

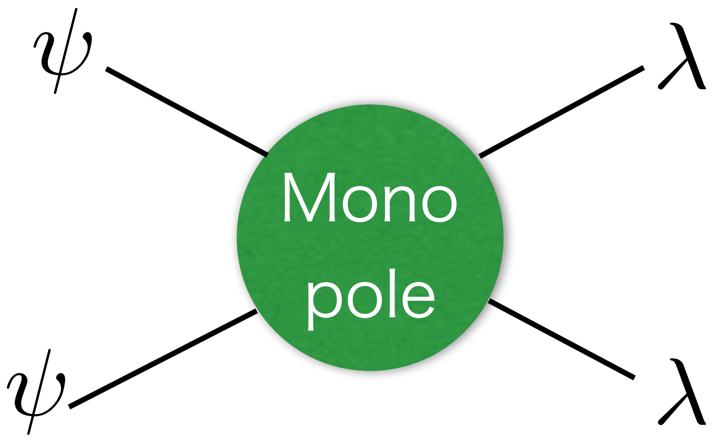
$$Spin(7) \to U(1)^3$$

3 fundamental monopoles

- 3-dimensional classical Coulomb branch
- Monopole generates non-perturbative effects and modifies the classical Coulomb branch drastically.

Monopole effects

Monopole(instanton) looks like a vertex for fermions.



$$\sim \lambda^2 \psi^2 e^{-S_{\text{on-shell}}}$$

If the monopole-vertex has two fermions, the non-perturbative effects appear in the superpotential.

$$\mathcal{L} \ni \int d^2\theta\, W_{\text{superpotential}} \ni \bar{\psi}\chi\phi^{\#}$$

Fate of Coulomb branch in pure SYM

Spin(7) has rank 3. Its CB is **classically** 3-dimensional.

$$Spin(7) \xrightarrow{Y_1, Y_2, Y_3} U(1)^3$$

Fermion zero-modes from gauginos

CB operator	gaugino
$Y_1 = \exp[\sigma_1 - \sigma_2]$	2
$Y_2 = \exp[\sigma_2 - \sigma_3]$	2
$Y_3 = \exp[2\sigma_3]$	2

σ_i : Adjoint scalars

Without matter, only gaugino contributes.

$$W = \sum_i \frac{1}{Y_i}$$

All the flat directions are lifted (no SUSY vacuum).

3d Spin(7) with spinor matters

- Fermion zero-modes

CB operator	gaugino	spinor
$Y_1 = \exp[\sigma_1 - \sigma_2]$	2	$1 + \mathrm{sign}(\sigma_1 - \sigma_2 - \sigma_3)$
$Y_2 = \exp[\sigma_2 - \sigma_3]$	2	0
$Y_3 = \exp[2\sigma_3]$	2	$1 - \mathrm{sign}(\sigma_1 - \sigma_2 - \sigma_3)$

$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$

Y2 is lifted. Y1 (or Y3) is lifted depending on the sign.

Globally, we need **one coordinate** for the Coulomb branch.

$$W = \frac{1}{Y_2} + \begin{cases} \frac{1}{Y_3} & (\sigma_1 > \sigma_2 + \sigma_3) \\ \frac{1}{Y_1} & (\sigma_1 < \sigma_2 + \sigma_3) \end{cases}$$

For all the regions, the quantum CB is described by $Z := Y_1 Y_2^2 Y_3$

$$\sim \exp[\sigma_1 + \sigma_2]$$

Exact superpotentials: Spin(7) with Ns spinors

Matter contents and moduli coordinates

	$Spin(7)$	$SU(N_S)$	$U(1)_S$	$U(1)_R$
S	$2^N = \mathbf{8}$	$\square$	1	R_S
λ	$\mathbf{Adj.}$	1	0	1
$M_{SS} := SS$	1	$\square\!\square$	2	$2R_S$
$B_S := S^4$	1	$\begin{array}{c}\square\\\square\\\square\\\square\end{array}$	4	$4R_S$
$Z := Y_1 Y_2^2 Y_3$	1	1	$-2N_S$	$-8-2N_S(R_S-1)$

← Matter contents

← Higgs branch coordinates

← Coulomb branch

$$W_{N_S\le 3} = \left(\frac{1}{Z\det M_{SS}}\right)^{\frac{1}{4-N_S}}$$

$$W_{N_S=4} = X\left[Z(\det M_{SS} - B_S^2) - 1\right]$$

X: Lagrange multiplier

$$W_{N_S=5} = Z\left(\det\ M_{SS} - B_S^i B_S^j M_{SS,ij}\right)$$

Exact superpotentials: Spin(7) with Ns spinors

$$W_{N_S \leq 3} = \left(\frac{1}{Z \det M_{SS}} \right)^{\frac{1}{4-N_S}}$$

→ Runaway potential (no SUSY vacuum)

$$W_{N_S=4} = X \left[Z(\det M_{SS} - B_S^2) - 1 \right]$$

→

$$Z(\det\ M_{SS} - B_S^2) = 1$$

Large vev of CB corresponds to small vevs of HB.

moduli space is highly deformed.

$$W_{N_S=5} = Z \left(\det\ M_{SS} - B_S^i B_S^j M_{SS,ij} \right)$$

→ S-confinement:

meson M, baryon B and a single gauge singlet Z

At the origin of moduli space, there is no symmetry breaking.

Nf vectors and Ns spinors

- Spin(7) allows **vector** and **spinor** representations. These zero-modes are

	adjoint	vector	spinor
Y_1	2	0	$1+\mathrm{sign}(\phi_1-\phi_3)$
Y_2	2	0	0
Y_3	2	2	$1-\mathrm{sign}(\phi_1-\phi_3)$

Y2 is still lifted while Y3 is not. Y1 depends on the sign.

$$W=\frac{1}{Y_2}+\begin{cases}0 & (\phi_1>\phi_3)\\ \frac{1}{Y_1} & (\phi_1<\phi_3)\end{cases}$$

One can generally expect two-dimensional CB.

$$Z:=Y_1Y_2^2Y_3$$

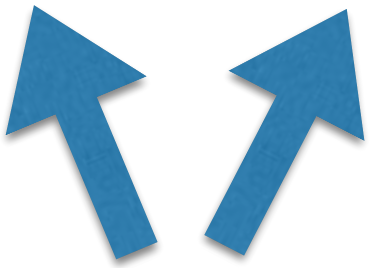
$$Y_{spin}:=Y_1^2Y_2^2Y_3 \quad (\phi_1>\phi_3)$$

[Aharony-Razamat-Seiberg-Willett '13]

S-confinement for (Nf, Ns)

We find s-confinement phases for

(Nf, Ns) = (0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (6, 0)



Today

3 vectors & 2 spinors

	$Spin(7)$	$SU(3)$	$SU(2)$	$U(1)_Q$	$U(1)_S$	$U(1)_R$
Q	$\square$	$\square$	1	1	0	R_f
S	$\mathbf{2^N = 8}$	1	$\square$	0	1	R_S
λ	**Adj.**	1	1	0	0	1
$M_{QQ} := QQ$	1	$\square\!\square$	1	2	0	$2R_f$
$M_{SS} := SS$	1	1	$\square\!\square$	0	2	$2R_S$
$P_{S,3} := SQ^3S$	1	1	$\square\!\square$	3	2	$3R_f + 2R_S$
$P_{A,1} := SQS$	1	$\square$	1	1	2	$R_f + 2R_S$
$P_{A,2} := SQ^2S$	1	$\bar{\square}$	1	2	2	$2R_f + 2R_S$
$Z := Y_1 Y_2^2 Y_3$	1	1	1	-6	-4	$-8 - 6(R_f - 1) - 4(R_S - 1)$
Y_{spin} for $\phi_1 \geq \phi_3$	1	1	1	-3	-4	$-5 - 3(R_f - 1) - 4(R_S - 1) = 2 - 3R_f - 4R_S$

← matter contents

← Higgs branch coordinates

The theory is s-confining and described by gauge singlets and the following superpotential.

$$W = Z\left(\det\ M_{QQ} \det\ M_{SS} - \det\ P_{S,3} + P_{A,2}^2 M_{QQ} - \frac{1}{2} P_{A,1}^2 M_{QQ}^2\right)$$
$$+ Y_{spin}\left(P_{A,1} P_{A,2} - M_{SS} P_{S,3}\right)$$

Spin(7)
(Nf,Ns)=(3,2)

dual

Non-gauge theory with
$M_{QQ}, M_{SS}, P_{S3}, P_{A1}, P_{A2}, Z$ and Y_{spin}

Vectors and one spinor: (Nf, Ns) = (4, 1)

	$Spin(7)$	$SU(4)_Q$	$U(1)_Q$	$U(1)_S$	$U(1)_R$
Q	$\mathbf{7}$	$\square$	1	0	R_f
S	$\mathbf{8}$	1	0	1	R_S
$\eta = \Lambda^b_{N_f,N_S}$	1	1	$2N_f$	$2N_S$	$2N_f(R_f-1)+2N_S(R_S-1)+10$
$M_{QQ} := QQ$	1	$\square\square$	2	0	$2R_f$
$M_{SS} := SS$	1	1	0	2	$2R_S$
$P := SQ^3S$	1	$\bar{\square}$	3	2	$3R_f + 2R_S$
$R := SQ^4S$	1	1	4	2	$4R_f + 2R_S$
$Z := Y_1Y_2^2Y_3$	1	1	-8	-2	$-8-2(R_S-1)-8(R_f-1) = 2-8R_f-2R_S$
$Y_{spin} := Y_1^2Y_2^2Y_3$	1	1	-8	-4	$-10-8(R_f-1)-4(R_S-1)$

← matter contents

← gauge invariants of the Higgs branch

- Semi-classically, two Coulomb branch operators are necessary: Z & Y
- But we find a quantum identification between them:

$$Z \sim Y_{spin} M_{SS}$$

$$W = Y_{spin}[M_{SS}^2 \det\, M_{QQ} + P^2 M_{QQ} - R^2]$$

Consistency checks

- Coincide with semi-classical analysis of the moduli space: okay
- Up lift to 4d N=1 SUSY gauge theories ← I will show this
- Flow to 3d N=2 G2/SU(3)/Spin(N < 7) gauge theories: okay
- Parity anomaly matching: okay
- Superconformal indices: consistent on both sides

3d and 4d relation

- In 4d, there is a chiral anomaly, but not for 3d.
- Additional U(1) symmetry in 3d
- In order to connect 3d and 4d, we need to introduce effects breaking the U(1) symmetry.
- On S1 x R3, additional instantons (known as KK-monopole or twisted instanton) should be included.
- KK-monopole only contains 2 gaugino zero-modes, which leads to

$$\Delta W = \eta Z$$

Back to the 4d

Up lift into 4d

CBs should be integrated out since they are gauge fields.

One must include effects from S1 x R3 (KK-monopole).

$$\Delta W = \eta Z$$

$$W_{N_S=5} = Z\left(\det\, M_{SS} - B_S^i B_S^j M_{SS,ij}\right) \quad +\eta Z$$

→

Integrating out Z

$$\frac{\partial W}{\partial Z} = 0$$

$$\det M_{SS} - B_S^2 M_{SS} + \eta = 0$$

4d constraint [Pouliot ‘95]

3d and 4d relation

Up lift to 4d: (Nf, Ns)=(4, 1) case

CB is integrated out.

One must include effects from S1 x R3 (KK - monopole)

$$W_{\mathbb{S}_1\times\mathbb{R}_3} = \eta Z \ \sim \eta Y_{spin} M_{SS}$$

η : dynamical scale of 4d gauge coupling

$$W = Y_{spin}[M_{SS}^2 \det\ M_{QQ} + P^2 M_{QQ} - R^2] + \eta Y_{spin} M_{SS}$$

Integrating-out Y (4d limit)

$$M_{SS}^2 \det\ M_{QQ} + P^2 M_{QQ} - R^2 + \eta M_{SS} = 0$$

4d quantum constraint

[P. Cho '97]

Summary & Discussion

- We studied 3d N=2 SUSY Spin(7) gauge theory and found some s-confining phases whose CB is 1 or 2 dimensional.
- The most of the CB is lifted and it depends on the matter contents, which could be different from the semiclassical picture in general.
- 3d & 4d are connected via KK-monopole.
- Spin(N) $(7 < N < 15)$ is possible but not so straightforward.
- For larger Ns, we expect some Seiberg dual description but now not known. The dual would be U(F-4) with symmetric matters.

Thank you for your attention.

“Superconformal Index”

- counting BPS states with weights
- partition function on S1 x S2

spin(3) rotation of S2

$$I = \mathrm{Tr}\left[(-1)^F e^{-\beta\{Q,S\}} x^{R+2j_3} t^F\right]$$

Fermion number

scalar BPS state (operator) with R-charge r & global U(1)charge q

➡ $x^r t^q$

- We can compute this quantity by employing a localization technique.

SCI in Spin(7) Ns=5

The theory is s-confining.

$$R_S = \frac{1}{8}$$

	$SU(5)$	$U(1)_S$	$U(1)_R$
$M_{SS} := SS$	$\square\square$	2	$2R_S = \frac{1}{4}$
$B_S := S^4$	$\bar{\square}$	4	$4R_S = \frac{1}{2}$
$Z := Y_1 Y_2^2 Y_3$	1	-10	$2 - 10R_S = \frac{3}{4}$

$\rightarrow 15u^2x^{1/4}$

$\rightarrow 5u^4x^{1/2}$

$\rightarrow \frac{x^{3/4}}{u^{10}}$

$$\overset{M_{SS}}{\downarrow}\quad \overset{\mathbf{15}\times\mathbf{15}|_{\text{symmetric}} = \mathbf{120}}{\overset{M_{SS}^2 + B_S}{\downarrow}} \quad \overset{Z}{\swarrow} \quad \overset{ZM_{SS}}{\downarrow}$$

$$1 + 15u^2x^{1/4} + 125u^4\sqrt{x} + \left(\frac{1}{u^{10}} + 755u^6\right)x^{3/4} + \left(3675u^8 + \frac{15}{u^8}\right)x + \left(15252u^{10} + \frac{125}{u^6}\right)x^{5/4}$$
$$+ \left(\frac{1}{u^{20}} + 55880u^{12} + \frac{750}{u^4}\right)x^{3/2} + 5\left(37004u^{14} + \frac{717}{u^2} + \frac{3}{u^{18}}\right)x^{7/4} + \left(562985u^{16} + \frac{125}{u^{16}} + 14402\right)x^2$$
$$+ \left(\frac{1}{u^{30}} + 1594185u^{18} + \frac{750}{u^{14}} + 50245u^2\right)x^{9/4} + \left(4241879u^{20} + 155550u^4 + \frac{3585}{u^{12}} + \frac{15}{u^{28}}\right)x^{5/2}$$
$$+ \left(10688125u^{22} + 433550u^6 + \frac{14403}{u^{10}} + \frac{125}{u^{26}}\right)x^{11/4} + \left(\frac{1}{u^{40}} + 25661515u^{24} + \frac{750}{u^{24}} + 1097955u^8 + \frac{50270}{u^8}\right)x^3 + \cdots$$

One can also derive the same index from $M_{SS},\ B_S,\ Z$

SCI in Spin(7) (Nf,Ns)=(4,1)

	$SU(4)_Q$	$U(1)_Q$	$U(1)_S$	$U(1)_R$	
$M_{QQ} := QQ$	$\square\square$	2	0	$2R_f$	$10t^2x^{1/4}$
$M_{SS} := SS$	1	0	2	$2R_S$	$u^2x^{1/4}$
$P := SQ^3S$	$\bar{\square}$	3	2	$3R_f + 2R_S$	$4t^3u^2x^{5/8}$
$R := SQ^4S$	1	4	2	$4R_f + 2R_S$	$t^4u^2x^{3/4}$
$Y_{spin} := Y_1^2Y_2^2Y_3$	1	-8	-4	$-10-8(R_f-1)-4(R_S-1)$	$\frac{x^{1/2}}{t^8u^4}$

The theory is dual to a non-gauge theory with M_{QQ}, M_{SS}, P, R and Y_{spin}

$Y_{spin}M_{SS}$

$$
\begin{aligned}
&1 + x^{1/4}\left(10t^2 + u^2\right) + \sqrt{x}\left(\frac{1}{t^8u^4} + 55t^4 + 10t^2u^2 + u^4\right) + 4t^3u^2x^{5/8} \\
&\quad + x^{3/4}\left(220t^6 + 56t^4u^2 + 10t^2u^4 + \frac{10t^2 + u^2}{t^8u^4} + u^6\right) + 4t^3u^2x^{7/8}\left(10t^2 + u^2\right) \\
&\quad + x\left(\frac{1}{t^{16}u^8} + 715t^8 + \frac{1}{t^8} + 230t^6u^2 + \frac{10}{t^6u^2} + 56t^4u^4 + \frac{55}{t^4u^4} + 10t^2u^6 + u^8\right) \\
&\quad + 4t^3u^2x^{9/8}\left(\frac{1}{t^8u^4} + 55t^4 + 10t^2u^2 + u^4\right) \\
&\quad + x^{5/4}\left(\frac{1}{t^{16}u^6} + \frac{10}{t^{14}u^8} + 2002t^{10} + 770t^8u^2 + \frac{u^2}{t^8} + 240t^6u^4 + \frac{10}{t^6} + 56t^4u^6 + \frac{55}{t^4u^2} + 10t^2u^8 + \frac{220}{t^2u^4} + u^{10}\right) + \cdots,
\end{aligned}
$$