Exploring 5D Supersymmetric RG Flows

(Work in progress with T.Dumitrescu,G.Festuccia,L.Ruggeri)

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Motivation

- Discovery of 5D and 6D SCFTs is a groundbreaking result emerging from String Theory [Witten; Strominger; Seiberg]

- In many aspects, these theories challenge our current understanding of Quantum Field Theory

- Important information can be obtained by analyzing supersymmetric protected quantities.

5D Supersymmetry

Three roads to five-dimensional UV superconformal fixed points:

- Field Theory Analysis [Seiberg+Intriligator-Morrison]
- M-theory on Calabi-Yau 3-fold [Morrison-Seiberg, Douglas-Katz-Vafa]
- Webs of (p,q)-fivebranes [Aharony-Hanany-Kol]

Recently, new advances in the program of geometric classification
[Del Zotto-Heckman-Morrison] [Jefferson-Katz-Kim-Vafa]

IR description given by a 5D $\mathcal{N}=1$ quantum field theory:

- Collection of 5D vector multiplets with gauge algebra G and hypermultiplets in representation **R**
- Vector multiplets in 5D have real scalars ϕ^i, at a generic point on the Coulomb branch the gauge group is broken to $U(1)^r$ with r=rank(G).

5D Supersymmetry

A 5D vector multiplet on a circle reduces to a 4D $\mathcal{N}=2$ vector multiplet:

$$\downarrow$$

low-energy dynamics on the Coulomb branch can be analyzed with a 5D prepotential:

$$\mathcal{F} = \frac{1}{2g^2} h_{ij}\phi^i\phi^j + \frac{k}{6} d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{12}\left(\sum_{e\in Root} |e\cdot\phi|^2 + \sum_j \sum_{w\in \boldsymbol{R}_j} |w\cdot\phi + m_i|^3\right)$$

With (h,d) given in terms of generators of G and $\mathbf{R}_j$ weights for hypers. Leads to a renormalized effective gauge coupling:

$$t(\phi)_{ij} = \frac{1}{(g^2_{\text{eff}})_{ij}} = \partial_i\partial_j\mathcal{F}$$

5D Supersymmetry

Example: A 5D SU(2) gauge theory with N_f quarks has:

$$t(\phi) = t_0 + 16\phi - \sum_{i=1}^{N_f} |\phi - m_i| - \sum_{i=1}^{N_f} |\phi + m_i|$$

For $m_i = 0$, when $N_f \leq 7$, the limit $t_0 \to 0$ is well defined:

$$t(\phi) = (16 - 2N_f)\phi \longrightarrow$$

5D UV fixed points with enhanced E_{N_f+1} global symmetry

Goal: Describe other protected quantities that can be used to explore the 5D RG flow

Supersymmetric Indices

A natural class of SUSY indices are defined by partition functions on manifolds with topology $\mathcal{M} = S^{D-1} \times S^1$:

- Count supersymmetric states on $S^{D-1} \times \mathbb{R}_{\text{time}}$,
- In a CFT, equivalent to a counting of BPS local operators in flat space.

[Kinney-Maldacena-Minwalla-Raju]
[Romelsberger]

A proper definition of non-conformal supersymmetric index is often possible but more subtle:

- How to preserve SUSY?
- New parameters induced by curved background?
- What does the index depend on?

Non-conformal indices are extremely important for analyzing the RG flow.

Curved Space Supersymmetry

In a supersymmetric theory $T_{\mu\nu}$ sits in a multiplet together with its bosonic and fermionic partners $\mathcal{J}_B^i$, $\mathcal{J}_F^i$

$$\Delta\mathscr{L} = \Delta g^{\mu\nu} T_{\mu\nu} + \sum_i (\mathcal{B}^i \mathcal{J}_B^i + \mathcal{F}^i \mathcal{J}_F^i) + \ldots$$

<u>Key point</u>: $(g_{\mu\nu}, \mathcal{B}^i)$ should be promoted to elements of an off-shell supergravity multiplet

[Festuccia-Seiberg]

Set $\mathcal{F}^i = 0$, a supercharge Q exists if:

$$\delta_Q \mathcal{F}^i = 0 \quad \longrightarrow \quad \nabla_\mu \zeta + M(g_{\mu\nu}, \mathcal{B}^i)\zeta = 0$$

Solutions to these equations characterize the supersymmetric background $(g_{\mu\nu}, \mathcal{B}^i)$

Previous Attempts

Two known proposals for a $\mathcal{N}=1$ non-conformal theory on $S^4 \times S^1$:

1. Use classical SUSY 5D CS conformal invariant action. Not valid for Sp(N) gauge groups
   → A Q-exact term can be constructed and localization procedure applied successfully [Kim-Kim-Lee]
2. Use a 5D SYM-like theory with a position dependent coupling
   → "We should think the theory has infinite coupling constant" [Terashima]

Localization seems very powerful for studying 5D superconformal index properties. We would like to have a better description of the non-conformal theory.

Non-Conformal N=1 Index

Any 5D $\mathcal{N}=1$ SCFT can be conformally mapped to a cylinder $S^4 \times \mathbb{R}$:

Idea: Look for a subalgebra of the 5d $\mathcal{N}=1$ SCA F(4) that does not involve any conformal transformation on $S^4 \times \mathbb{R}$

Largest superalgebra possibly realized is $\mathfrak{su}(2|1) \subset$ F(4):

$$\{Q^{+}{}_{\alpha}, Q^{-\beta}\} = 4\delta_{\alpha}{}^{\beta}(H-R) + 4\,M_{\alpha}{}^{\beta}$$

- $\mathrm{SU}(2)_r \times \mathrm{SU}(2)_l$ subgroup of SO(5) with generators $M_{\alpha}{}^{\beta}, \widetilde{M}^{\dot{\alpha}}{}_{\dot{\beta}}$
- $Q^{+}{}_{\alpha}$,$Q^{-\beta}$ are $\mathrm{SU}(2)_r$ invariant supercharges
- H , translations along cylinder
- R, Cartan of $\mathrm{SU}(2)_R$, R-symmetry is broken to U(1)

Curved Background

Look for a supergravity background which realizes the $\mathfrak{su}(2|1)$ SUSY algebra.

All 5D theories with SU(2) R-symmetry have a "Sonhius Multiplet":

$$(C, \psi^i_\alpha, X^{ij}, W_{\mu\nu}, R^{ij}_\mu, S^i_{\mu\alpha}, J_\mu, T_{\mu\nu})$$

Background fields are part of an off-shell supergravity multiplet:

$$\Delta\mathscr{L} = \Delta g^{\mu\nu} T_{\mu\nu} + \mathcal{A}^\mu J_\mu + (V^\mu)^{ij} R_{\mu\, ij} + \mathcal{V}^{\mu\nu} W_{\mu\nu}$$

$$+ t^{ij} X_{ij} + \mathcal{J}_C C + (\text{fermions})$$

Setting the variation of supergravity fermions to zero gives:

$$\nabla_\mu \zeta_i = t_i^{\ j} \Gamma_\mu \zeta_j + \frac{1}{2} \mathcal{F}_{\mu\nu} \Gamma^\nu \zeta_i + \frac{1}{2} \mathcal{P}^{\nu\rho} \Gamma_{\mu\nu\rho} \zeta_i$$

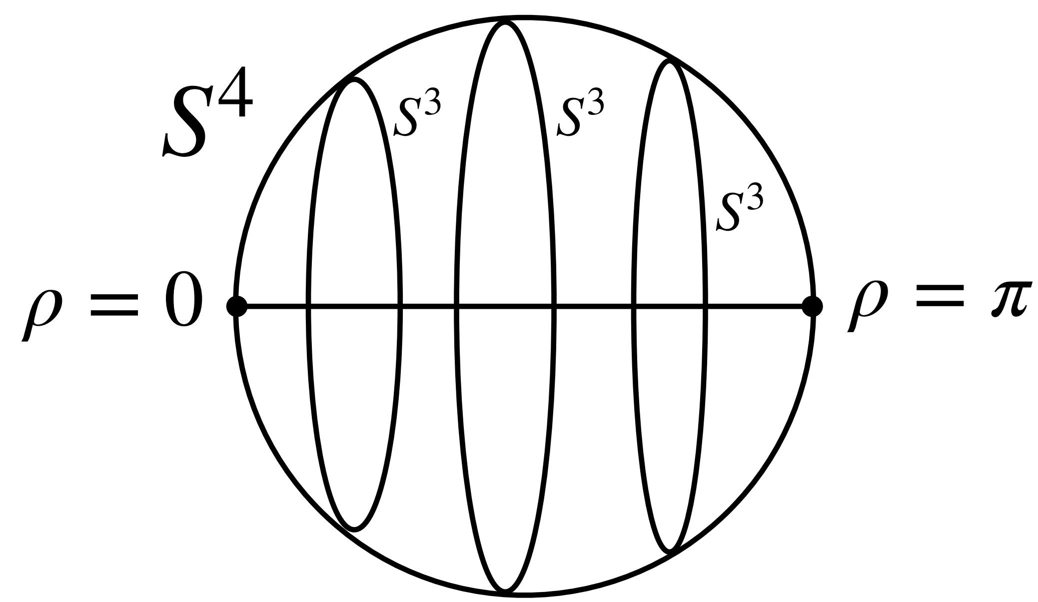
$$\mathcal{F} = d\mathcal{A}$$

$$\mathcal{P} \equiv \mathcal{V} - \mathcal{F}$$

Curved Background

There is a solution for a 4-sphere of radius ℓ with round metric:

$$ds^2 = -dt^2 + \ell^2(d\rho^2 + \sin^2(\rho)d\Omega_3) \qquad 0 \leq \rho \leq \pi$$



Background fields:

$$\mathcal{A} = -\frac{1}{2}\cos\rho\, dt \qquad t = \frac{1}{2\ell}\cos\rho\,(\sigma_3) \qquad V = \frac{i}{2\ell}(\sigma_3)\, dt \quad \mathcal{P} = 0$$

Comments

- A peculiar feature of this background is that $\mathcal{A}$ and t introduce position dependent couplings.

- It is only possible to realize the $\mathfrak{su}(2|1)$ SUSY algebra with 4 curved supercharges

- The most refined index we can define is:

$$I_{(5D)} = Tr_{\mathcal{H}}(-1)^F e^{-(\beta\Delta + \gamma_1 H + \gamma_2 \widetilde{\mathcal{M}}^{\mathrm{1}}{}_{\mathrm{1}} + i\sum_i F_i m_i)} \mathfrak{q}^k$$

  - $\Delta = \{Q_1{}^+, Q^{-1}\}$
  - F_i Cartan generators for global symmetries
  - k Instanton Charge

- For a conformal theory this agrees with 5D SCI of [Kim-Kim-Lee]

Non-Decoupling

Usually we do not expect the index to depend on continuous parameters. Consider a free hypermultiplet of mass m:

For $m \to \infty$ we expect a fully gapped IR theory with trivial index $I_{(5D)} = 1$.

Position dependent couplings introduce a new phenomenon:

Hypermultiplet mass becomes a function $m^2(\rho)$ of latitude angle on S^4, at both poles there are localized states with $E \sim 1/\ell$

Schur Background

- Reducing along the Hopf fiber of S^4 gives a supergravity background for a 4D $\mathcal{N}=2$ QFT on $S^3 \times \mathbb{R}$ known as "Schur Background" [SCFT Schur index: Gadde-Rastelli-Razamat-Yan]

- Using this background it is possible to define consistently the Schur index I_{Schur} for a non-conformal 4D $\mathcal{N}=2$ theory. [Dumitrescu-Festuccia]

- There is an intriguing relation between I_{Schur} and the spectrum of BPS particles on the Coulomb branch. [Cordova-Shao + Gaiotto]

- The 5D IR index contains information about spectrum of 5D BPS particles. Connection with [Iqbal-Vafa]

Applications

- 5D theories have operators charged under the conserved topological current $J_{Top} \sim * Tr(F \wedge F)$ called “Instanton Operators”

- Instanton operators participate in UV global symmetry enhancement. [Tachikawa]

- New 5D Index background allows for various instanton configurations. A new way to understand results of [Tachikawa]

- We can easily generalize to new 5D squashed backgrounds with a generic bounded function $f(\rho)$. Also possible to add BPS line defects and study their IR $\longrightarrow$ UV correspondence.

Conclusions

- We presented a new $S^4 \times S^1$ index $I_{(5D)}$ which is well defined for non-conformal 5D $\mathscr{N} = 1$ theories.

- Background fields make 5D IR dynamics much more rich.

- The index can be used to shed light on 5D SUSY RG flows by analyzing BPS particles and instanton operators.