~ Extended Gauge Theory Deformations from Flux Backgrounds ~

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Based on the joint work JHEP06(2018)13 with

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FNSNF
Swiss National Science Foundation

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

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0.0 Our motivation

- String theory is a good laboratory for various gauge theories
- Stringy realization of some supersymmetric gauge theories

[e.g. Witten '97]

0.1 Our method = probe brane

- Construct different gauge theories from the same <u>deformed</u> background.

  - where to embed a brane, dim. of the brane

  - a web via S/T-duality

  - ~~old-fashioned but~~ simple

  - understand microscopic descriptions

Recently, in another group,

[Choi, Fernández-Melgarejo, Sugimoto '17,'18]

0.2 Take-home abstract

☆Supersymmetric Lagrangians on curved space

using *RR-flux* backgrounds,

⊃ *twisted* covariant derivatives

⊃ have symmetries reflecting effect of *defect(s)?*

which are engineered via ______ *-deformation*

☆Sticked to the description.

0.2 Take-home abstract

☆Supersymmetric Lagrangians on curved space

using *RR-flux* backgrounds,

⊃ *twisted* covariant derivatives

⊃ have symmetries reflecting effect of *defect(s)?*

which are engineered via Ω-*deformation*

☆Sticked to the description.

1.1. Introduction: Ω-deformation

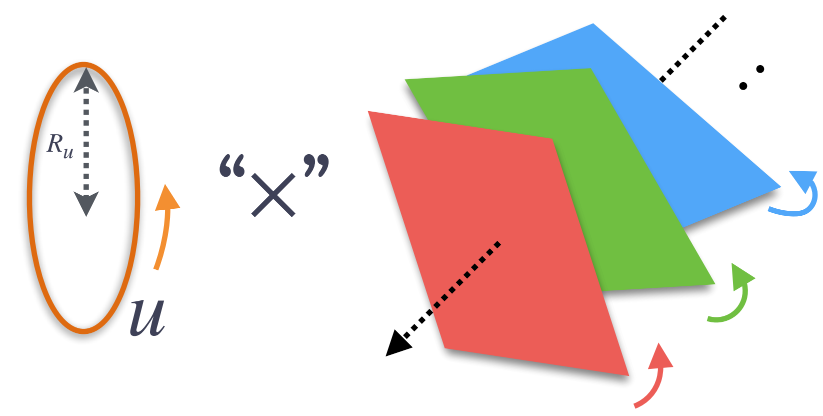
- **Calculation of Seiberg-Witten prepotentials** [Nekrasov '02]
- **Some correspondences**
  - **4d-2d** [Alday, Giotto, and Tachikawa '10]
  - **Gauge/Bethe** [Nekrasov, Shatashvili '09]
- **Topological amplitudes** [Antoniadis, Hohenegger, Narain, Taylor '10, Moskovic and Assi '16] [Angelantonj, Ignatios, Samsonyan '17]

etc…

1.1. Introduction: Ω-deformation

- **Calculation of Seiberg-Witten prepotentials** [Nekrasov '02]

$$\begin{cases} R\,u \simeq R\,u + 2\pi R \\ \theta_i \simeq \theta_i + \varepsilon_i\, R u \end{cases}$$



1.2. Introduction: "The string theory of Ω-deformation"

[Hellerman, Orlando, and Reffert '11, '12]

T-dual of Ω-background:

Start from 10d locally flat bgr:

$$ds_{10}^2 = -(dx^0)^2 + \sum_{i=1}^{4} dx^i dx^i + du^2 + \sum_{j=1}^{2} d\rho_j^2 + \rho_j^2 \theta_j^2$$

no Φ

no B_2

1.2. Introduction: "The string theory of Ω-deformation"

[Hellerman, Orlando, and Reffert '11, '12]

T-dual of Ω-background:

Do Ω-deformation (e.g. 2 ε_i 's)

$$ds_{10}^2 = -(dx^0)^2 + \sum_{i=1}^{4} dx^i dx^i + du^2 + \sum_{j=1}^{2} d\rho_j^2 + \rho_j^2 d(\theta_j + \varepsilon_j u)^2$$

1.2. Introduction: "The string theory of Ω-deformation"

[Hellerman, Orlando, and Reffert '11, '12]

T-dual of Ω-background:

T-dualize in u:

$$ds_{10}^2 = -(dx^0)^2 + \sum_{i=1}^{4} dx^i dx^i + \sum_{j=1}^{2} (d\rho_j^2 + \rho_j^2 d\theta_j^2) + \frac{d\tilde{u}^2 - (\varepsilon_1 \rho_1^2 d\theta_1 + \varepsilon_2 \rho_2^2 d\theta_2)^2}{1 + \varepsilon_1^2 \rho_1^2 + \varepsilon_2^2 \rho_2^2}$$

$$B_2 = \frac{\varepsilon_1 \rho_1^2 d\theta_1 + \varepsilon_2 \rho_2^2 d\theta_2}{1 + \varepsilon_1^2 \rho_1^2 + \varepsilon_2^2 \rho_2^2} \wedge d\tilde{u}$$

$$\Phi = -\frac{1}{2} \ln(1 + \varepsilon_1^2 \rho_1^2 + \varepsilon_2^2 \rho_2^2)$$

non-zero Φ and B_2

1.2. Introduction: "The string theory of Ω-deformation"

[Hellerman, Orlando, and Reffert '11, '12]

T-dual of Ω-background:

In rectangular coords:

$$ds_{10}^2 = -(dx^0)^2 + \sum_{i=1}^{4} dx^i dx^i + \sum_{j=6}^{9} dx^I dx^I$$

$$+ \frac{d\tilde{u}^2 - U_I U_J dx^I dx^J}{1 + U^K U_K}$$

$$B_2 = \frac{U_I dx^I}{1 + U^J U_J} \wedge d\tilde{u}$$

$$\Phi = -\frac{1}{2} \ln(1 + U^I U_I)$$

$$U_I dx^I = \varepsilon_1 (x^6 dx^7 - x^7 dx^6) + \varepsilon_2 (x^8 dx^9 - x^9 dx^8)$$
$$= \frac{1}{2} \omega_{IJ} x^I dx^J$$

ω_{IJ} **: anti-symmetric matrix**

1.2. Introduction: "The string theory of Ω-deformation"

[Hellerman, Orlando, and Reffert '11, '12]

Interesting facts:

1. Supersymmetry is preserved when

$$\omega_{IJ} = \pm(\star\omega)_{IJ} \Leftrightarrow \varepsilon_1 = \pm\varepsilon_2$$

1.2. Introduction: "The string theory of Ω-deformation"

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Interesting facts:

1. Supersymmetry is preserved when $\omega_{IJ} = \pm(\star\omega)_{IJ} \Leftrightarrow \varepsilon_1 = \pm\varepsilon_2$

2. Different ways of embedding lead to different types of gauge theories

(i) $\mathcal{E}_i$ turned on along the brane: Poincarè invariance broken

(ii) $\mathcal{E}_i$ turned on transverse to the brane: mass deformed Lagrangian

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2. Different ways of embedding lead to different types of gauge theories

[Hellerman, Orlando, and Reffert '11]

(i) ε_i turned on along the brane: Poincarè invariance broken

(ii) ε_i turned on transverse to the brane: mass deformed Lagrangian

Today: ε_i **NOT** on the probe brane

but we obtain a **different** type of gauge theories (not mass-deformed)

Contents :

1. Introduction ✔
2. Geometric engineering
3. Supersymmetric brane actions
4. Thoughts
5. Conclusion

2. Geometric Engineering

2. Geometric Engineering of RR flux backgrounds

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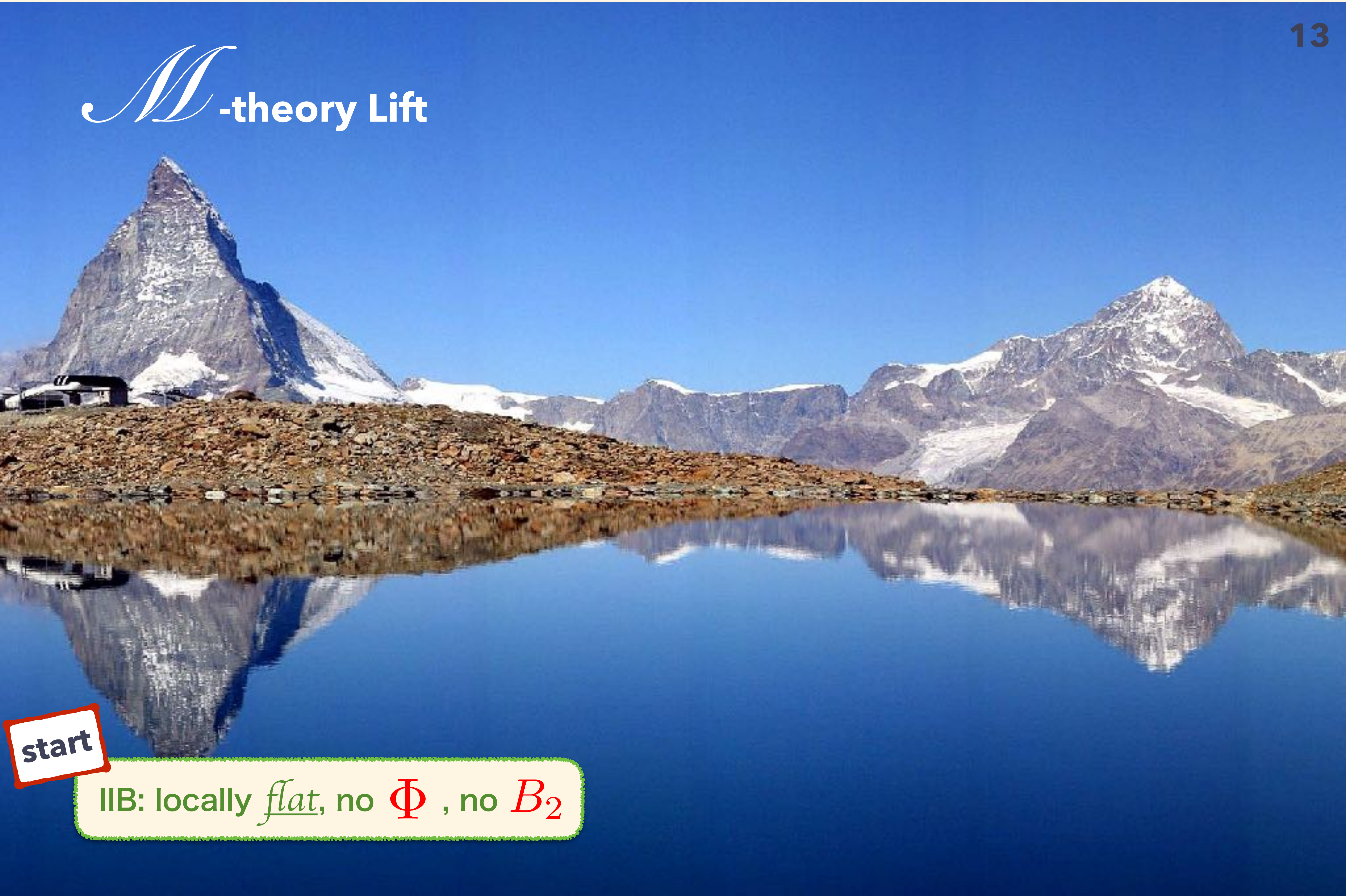
Point:

```
In[1]=  NSNS 3-form flux in IIA
Out[1]=  RR  3-form field in IIA
```

2.1 Road to RR-flux backgrounds

[Hellerman, Orlando, and Reffert '11, '12]
[Lambert, Orlando, and Reffert '13,'14]
[Lambert, Orlando, Reffert, and Y.S. '18]

$\mathscr{M}$-theory Lift



start

IIB: locally *flat*, no Φ , no B_2

2.1 Road to RR-flux backgrounds

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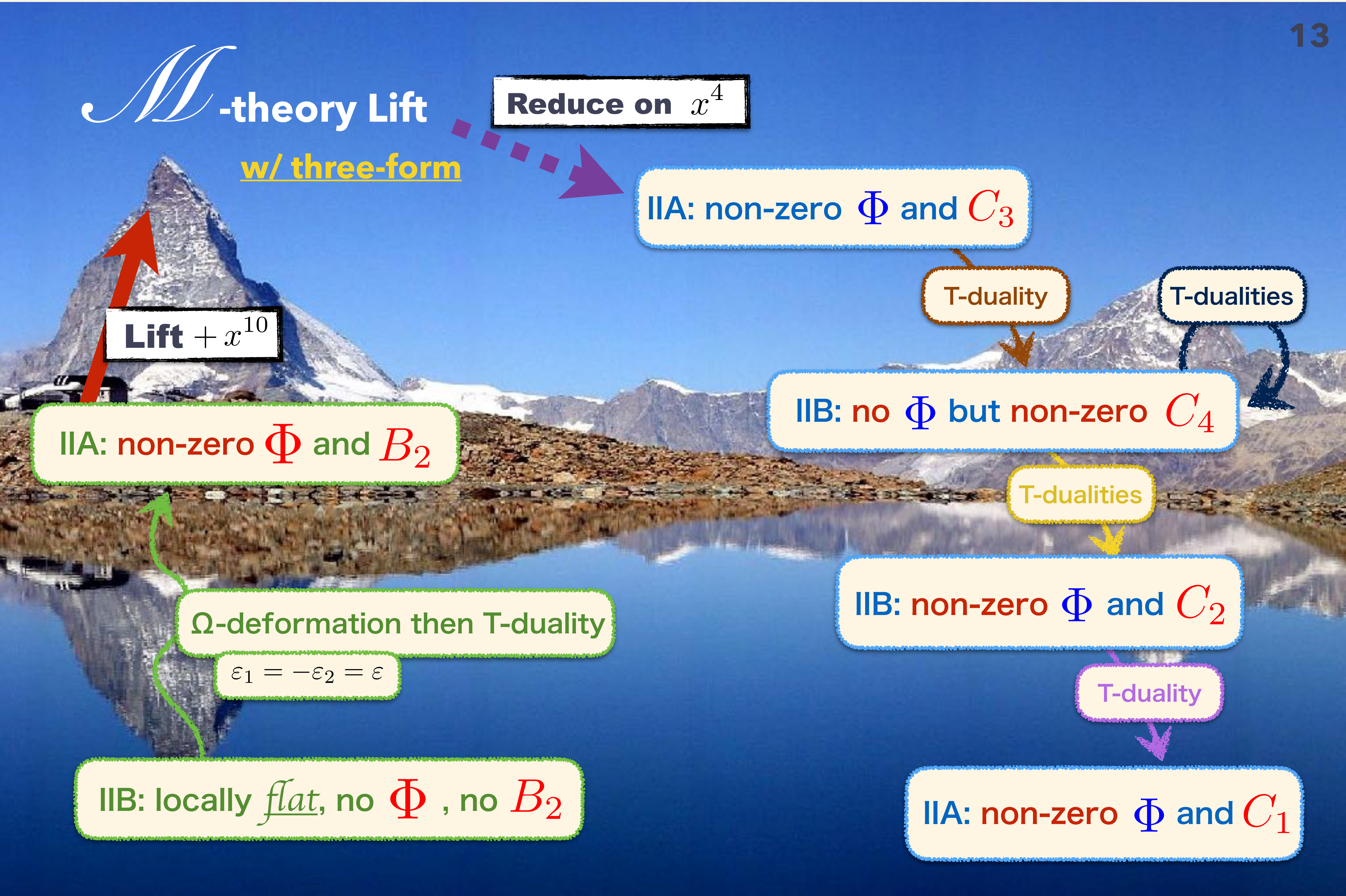

2.2. Probe branes chosen

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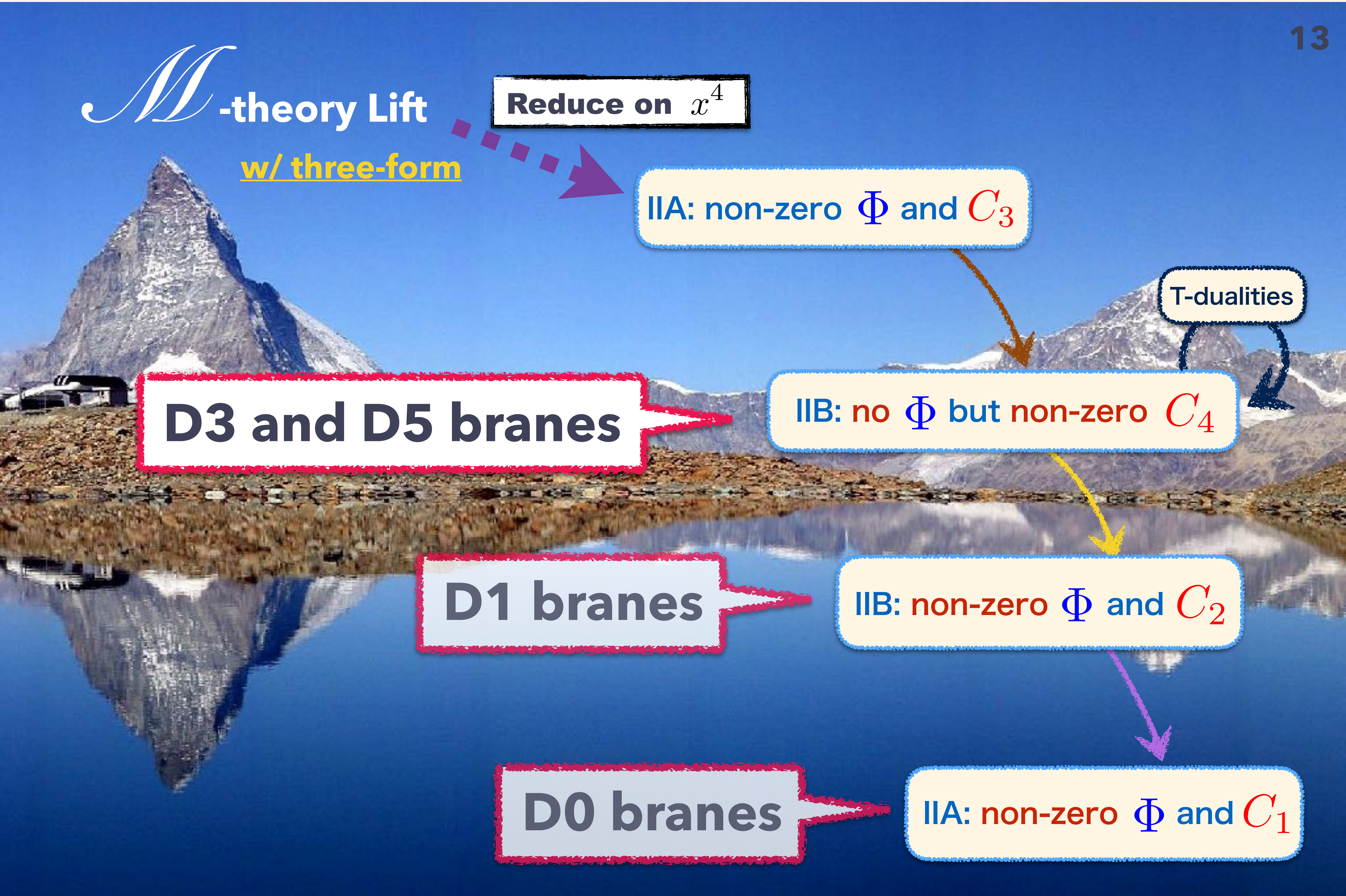

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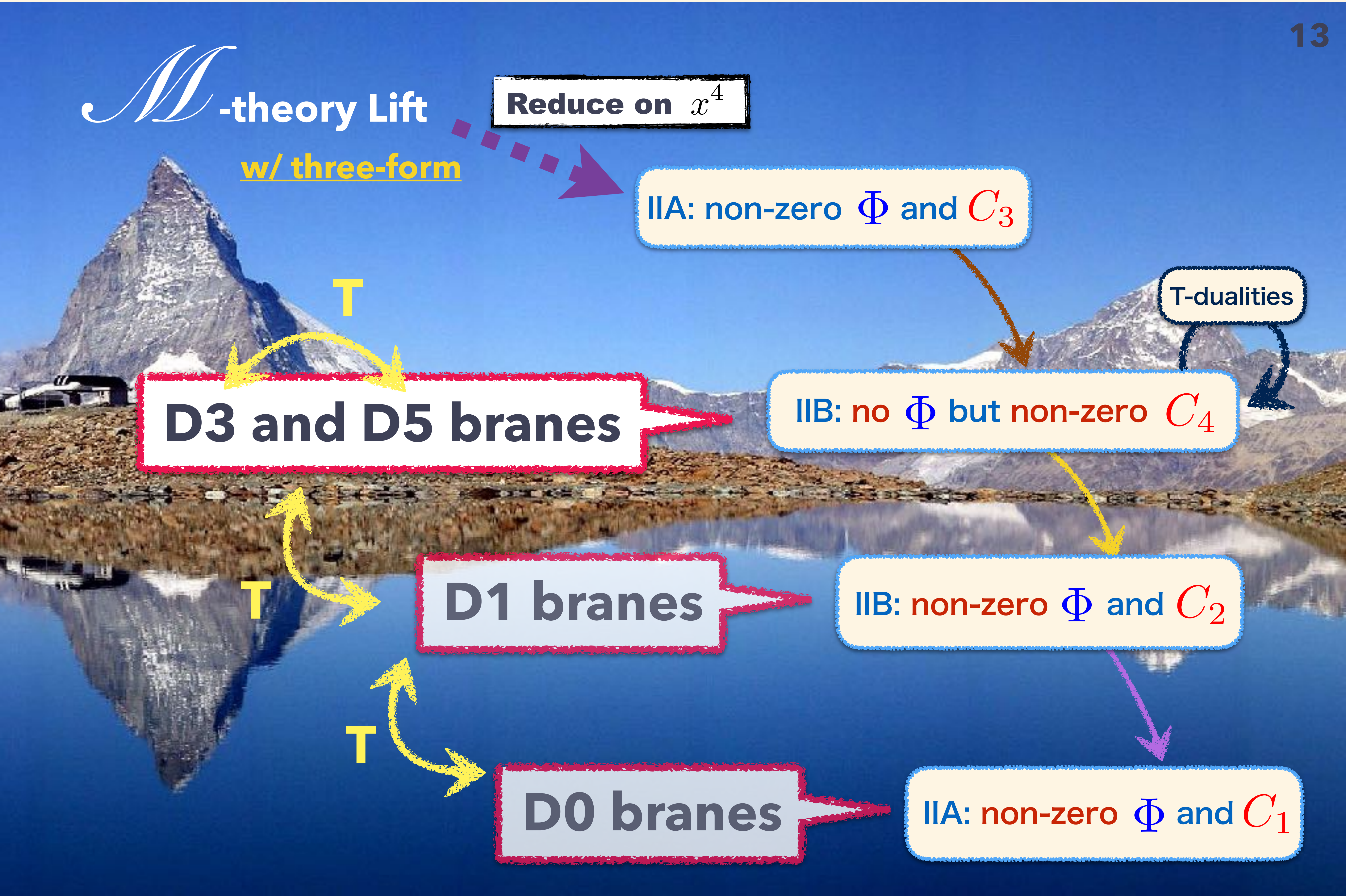

2.4.1 Representative: Background w/ RR four-form (for D5)

Index notation: $(x^0, x^1, x^5, x^2, x^3, x^4, x^6, x^7, x^8, x^9)$ with α, β for (x^0, x^1, x^5); a, b for (x^2, x^3, x^4); I, J for (x^6, x^7, x^8, x^9)

$\eta_{\alpha\beta} = \mathrm{diag}(-,+,+)$

$$ds_{10}^2 = g_{\alpha\beta}dx^\alpha dx^\beta + g_{ab}dx^a dx^b + g_{IJ}dx^I dx^J$$

$$C_4 = U_I dx^I \wedge (-dx^0 \wedge dx^1 \wedge dx^5 + \Delta^{-2} dx^2 \wedge dx^3 \wedge dx^4)$$

2.4.1 Backgrounds w/ RR four-form (for D5)

Index notation: $(x^0, x^1, x^5, x^2, x^3, x^4, x^6, x^7, x^8, x^9)$ — α, β: (x^0, x^1, x^5); a, b: (x^2, x^3, x^4); I, J: (x^6, x^7, x^8, x^9)

$\eta_{\alpha\beta} = \mathrm{diag}(-,+,+)$

$$ds_{10}^2 = \Delta \eta_{\alpha\beta} dx^\alpha dx^\beta + \Delta^{-1} \delta_{ab} dx^a dx^b + (\Delta \delta_{IJ} - \Delta^{-1} U_I U_J) dx^I dx^J$$

$$C_4 = U_I dx^I \wedge (-dx^0 \wedge dx^1 \wedge dx^5 + \Delta^{-2} dx^2 \wedge dx^3 \wedge dx^4)$$

$$\Delta = \sqrt{1 + U^I U_I} = \sqrt{1 + \varepsilon^2 x^I x_I}$$

$$U_I = \frac{1}{2} \omega_{JI} x^J$$

2.4.1 Backgrounds w/ RR four-form (for D5)

Index notation:

$$\underbrace{x^0, x^1, x^5}_{\alpha,\beta}, \underbrace{x^2, x^3, x^4}_{a,b}, \underbrace{x^6, x^7, x^8, x^9}_{I,J}$$

$\eta_{\alpha\beta} = \mathrm{diag}(-,+,+)$

$$ds^2_{10} = \Delta \eta_{\alpha\beta} dx^\alpha dx^\beta + \Delta^{-1}\delta_{ab} dx^a dx^b + (\Delta\delta_{IJ} - \Delta^{-1} U_I U_J) dx^I dx^J$$

$$C_4 = -\frac{1}{3!}\omega_{IJ} x^J dx^I \wedge \left(\Xi_{\alpha\beta\gamma} dx^\alpha \wedge dx^\beta \wedge dx^\gamma + \Delta^{-2}\Xi_{abc} dx^a \wedge dx^b \wedge dx^c\right)$$

$$\Xi_3 = \frac{1}{2}(-dx^0 \wedge dx^1 \wedge dx^5 + dx^2 \wedge dx^3 \wedge dx^4)$$

2.4.1 Backgrounds w/ RR four-form (for D5)

At $\mathcal{O}(\varepsilon)$, the metric becomes flat. So simple.

$$\Delta = \sqrt{1+\varepsilon^2 x^I x_I} \to 1$$

$$ds^2_{10} = \Delta\eta_{\alpha\beta}dx^\alpha dx^\beta + \Delta^{-1}\delta_{ab}dx^a dx^b + (\Delta\delta_{IJ} - \Delta^{-1}U_I U_J)dx^I dx^J$$

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At $\mathcal{O}(\varepsilon)$, the metric becomes flat. So simple.

$$\Delta = \sqrt{1+\varepsilon^2 x^I x_I} \to 1$$

$$ds_{10}^2 = -(dx^0)^2 + (dx^1)^2 + \cdots + (dx^9)^2$$

$$C_4 = -\frac{1}{3!}\omega_{IJ} x^J dx^I \wedge (\Xi_{\alpha\beta\gamma} dx^\alpha \wedge dx^\beta \wedge dx^\gamma + \Xi_{abc} dx^a \wedge dx^b \wedge dx^c)$$

$$\omega_{IJ} = 2\begin{pmatrix} 0 & -\varepsilon & 0 & 0 \\ \varepsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon \\ 0 & 0 & -\varepsilon & 0 \end{pmatrix}$$

$$\Xi_3 = \frac{1}{2}(-dx^0 \wedge dx^1 \wedge dx^5 + dx^2 \wedge dx^3 \wedge dx^4)$$

2.4.1 Backgrounds w/ RR five-form flux (for D5)

At $\mathcal{O}(\varepsilon)$, the metric becomes flat. So simple.

$$\Delta = \sqrt{1+\varepsilon^2 x^I x_I} \to 1$$

$$ds_{10}^2 = -(dx^0)^2 + (dx^1)^2 + \cdots + (dx^9)^2$$

↓ anti-self-dual for $\star_4$

anti-self-dual for $\star_6$

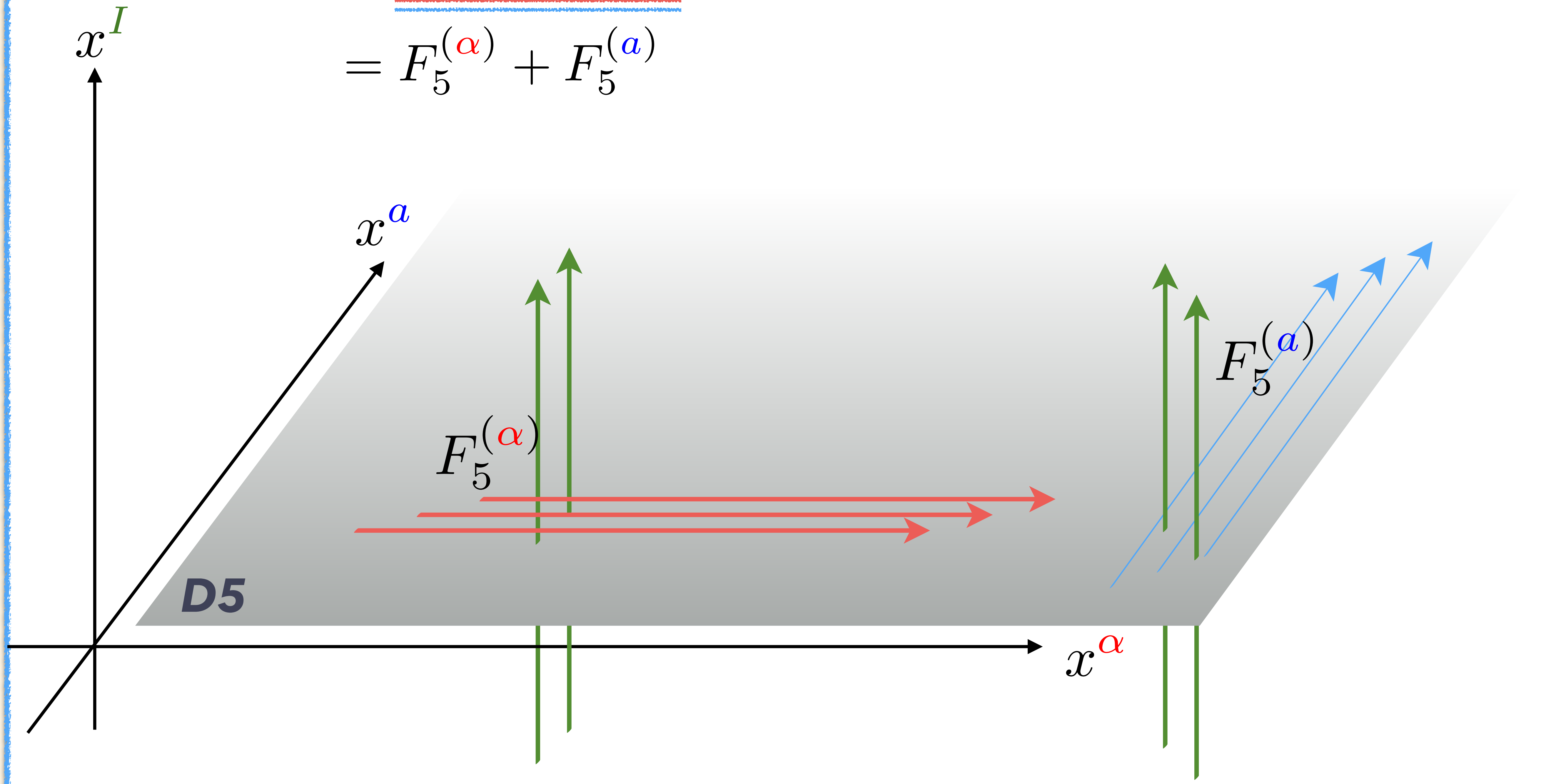
$$F_5 = \frac{1}{3!}\omega_{IJ} dx^I \wedge dx^J \wedge (\Xi_{\alpha\beta\gamma} dx^\alpha \wedge dx^\beta \wedge dx^\gamma + \Xi_{abc} dx^a \wedge dx^b \wedge dx^c)$$

$$\Xi_3 = \frac{1}{2}(-dx^0 \wedge dx^1 \wedge dx^5 + dx^2 \wedge dx^3 \wedge dx^4)$$

2.4.2 D5 probe brane in $(x^{\alpha}, x^{a}) = (x^0, \ldots, x^5)$

$$F_5 = \frac{1}{3!}\omega_{IJ} dx^I \wedge dx^J \wedge (\Xi_{\alpha\beta\gamma} dx^{\alpha} \wedge dx^{\beta} \wedge dx^{\gamma} + \Xi_{abc} dx^a \wedge dx^b \wedge dx^c)$$

$$= F_5^{(\alpha)} + F_5^{(a)}$$

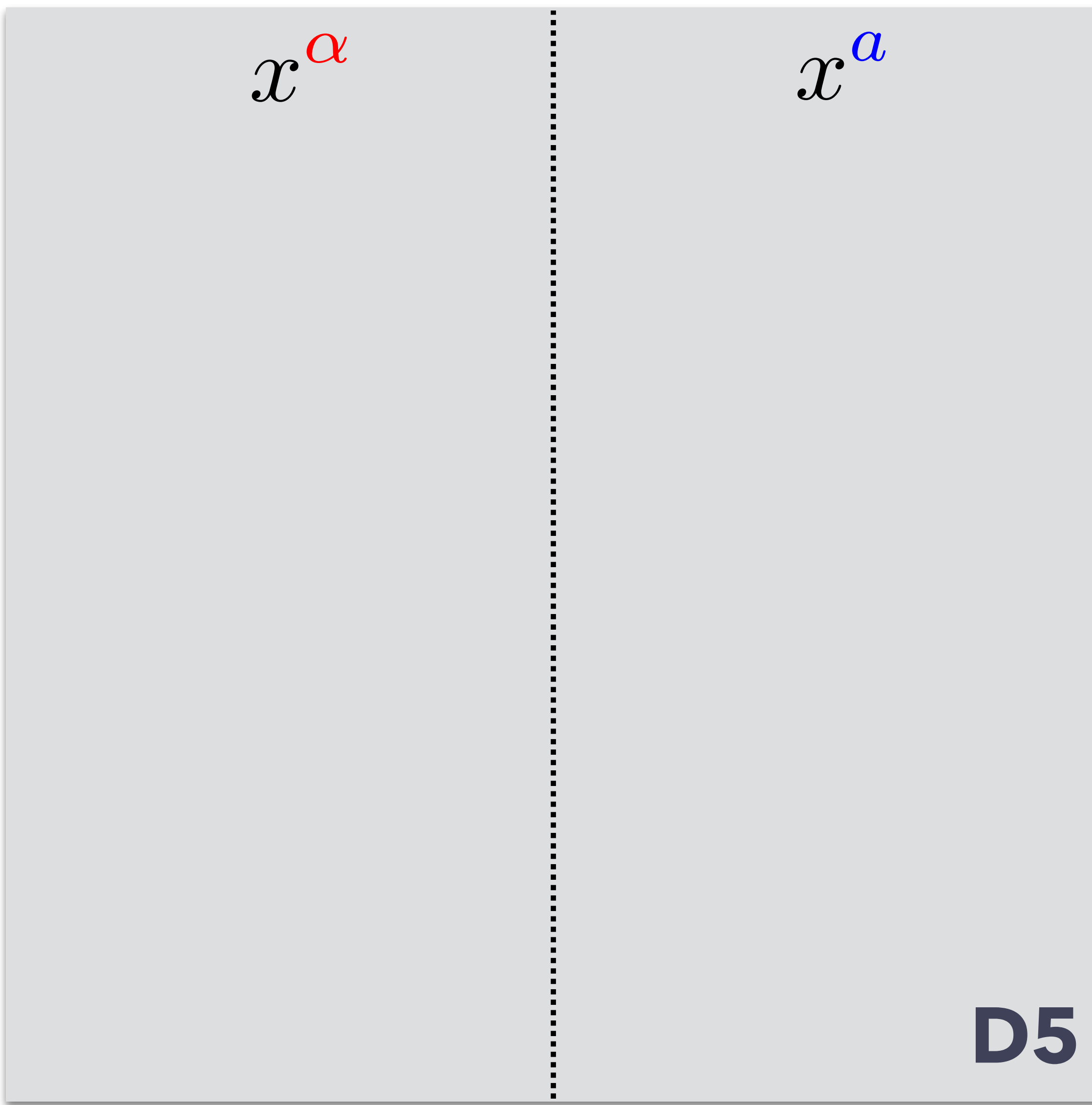


2.4.2 Expect three-dim defect"s" on D5

$$F_5 = \frac{1}{3!}\omega_{IJ} dx^I \wedge dx^J \wedge (\Xi_{\alpha\beta\gamma} dx^\alpha \wedge dx^\beta \wedge dx^\gamma + \Xi_{abc} dx^a \wedge dx^b \wedge dx^c)$$

$$= F_5^{(\alpha)} + F_5^{(a)}$$

More schematically (confusing)



☆ **a probe D5 brane in** $(x^0, \cdots, x^5)$

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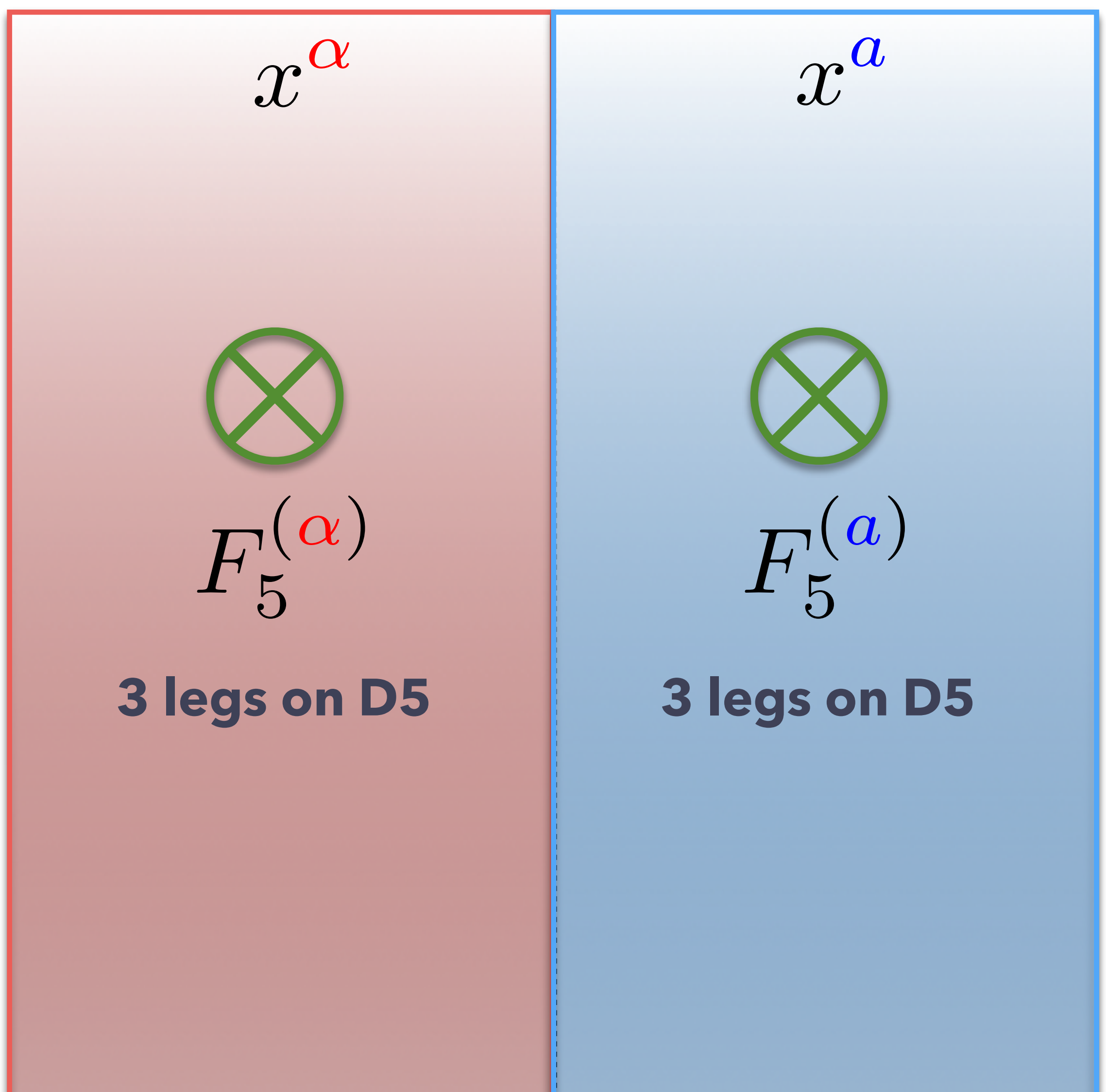
$$= F_5^{(\alpha)} + F_5^{(a)}$$

More schematically,



☆ **a probe D5 brane in** $(x^0, \cdots, x^5)$

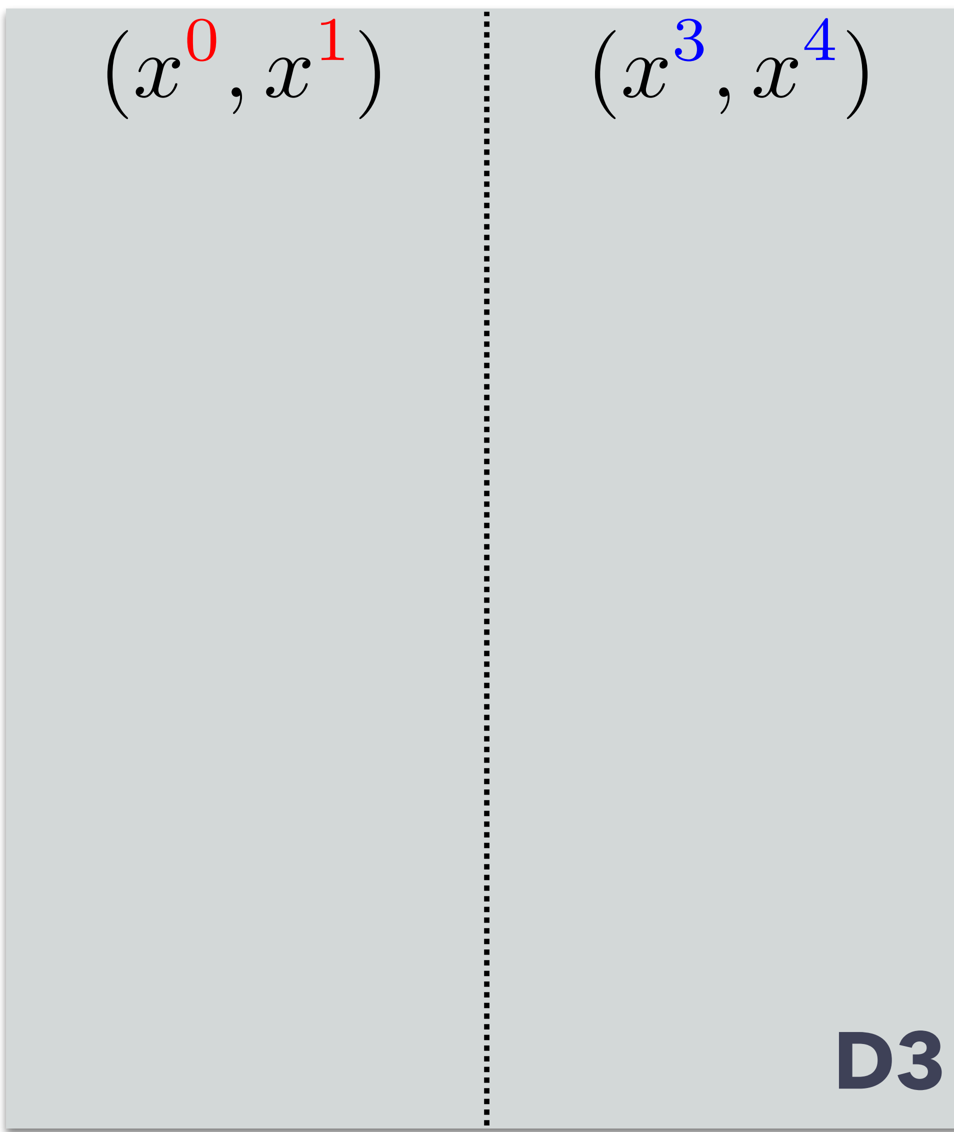
→6d susy gauge theory
w/ three-dim. defects

2.4.3 Expect two-dim. defect"s" on D3

$$F_5 = \frac{1}{3!}\omega_{IJ} dx^I \wedge dx^J \wedge (dx^0 \wedge dx^1 \wedge dx^2 + dx^3 \wedge dx^4 \wedge dx^5)$$

$$= F_5^{(\alpha)} + F_5^{(a)}$$

dx^2, dx^5 **: not on D3**



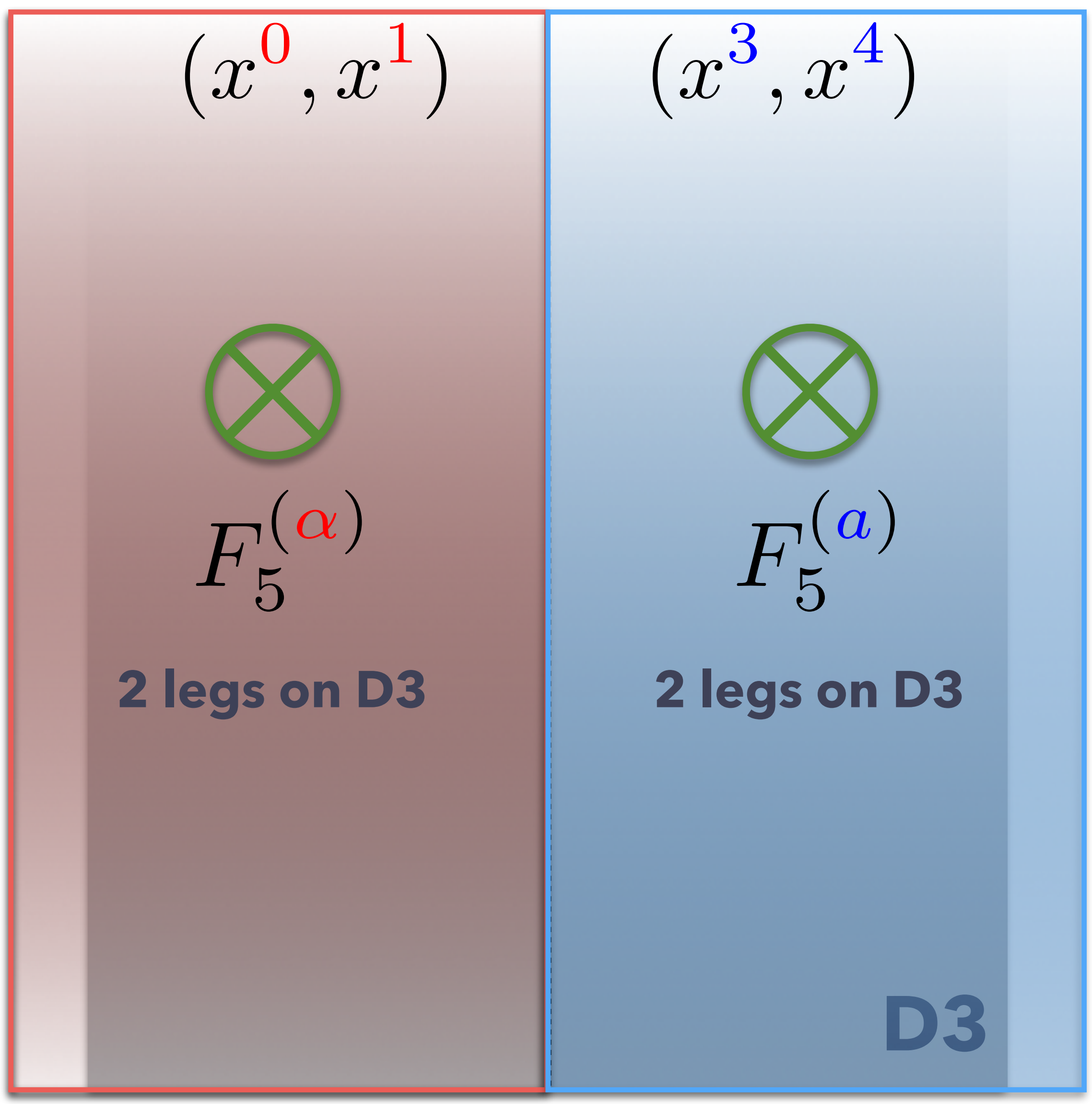
☆ **a probe D3 brane in** (x^0, x^1, x^3, x^4)

2.4.3 Expect two-dim. defect"s" on D3

$$F_5 = \frac{1}{3!}\omega_{IJ}dx^I \wedge dx^J \wedge (dx^0 \wedge dx^1 \wedge dx^2 + dx^3 \wedge dx^4 \wedge dx^5)$$

$$= F_5^{(\alpha)} + F_5^{(a)}$$

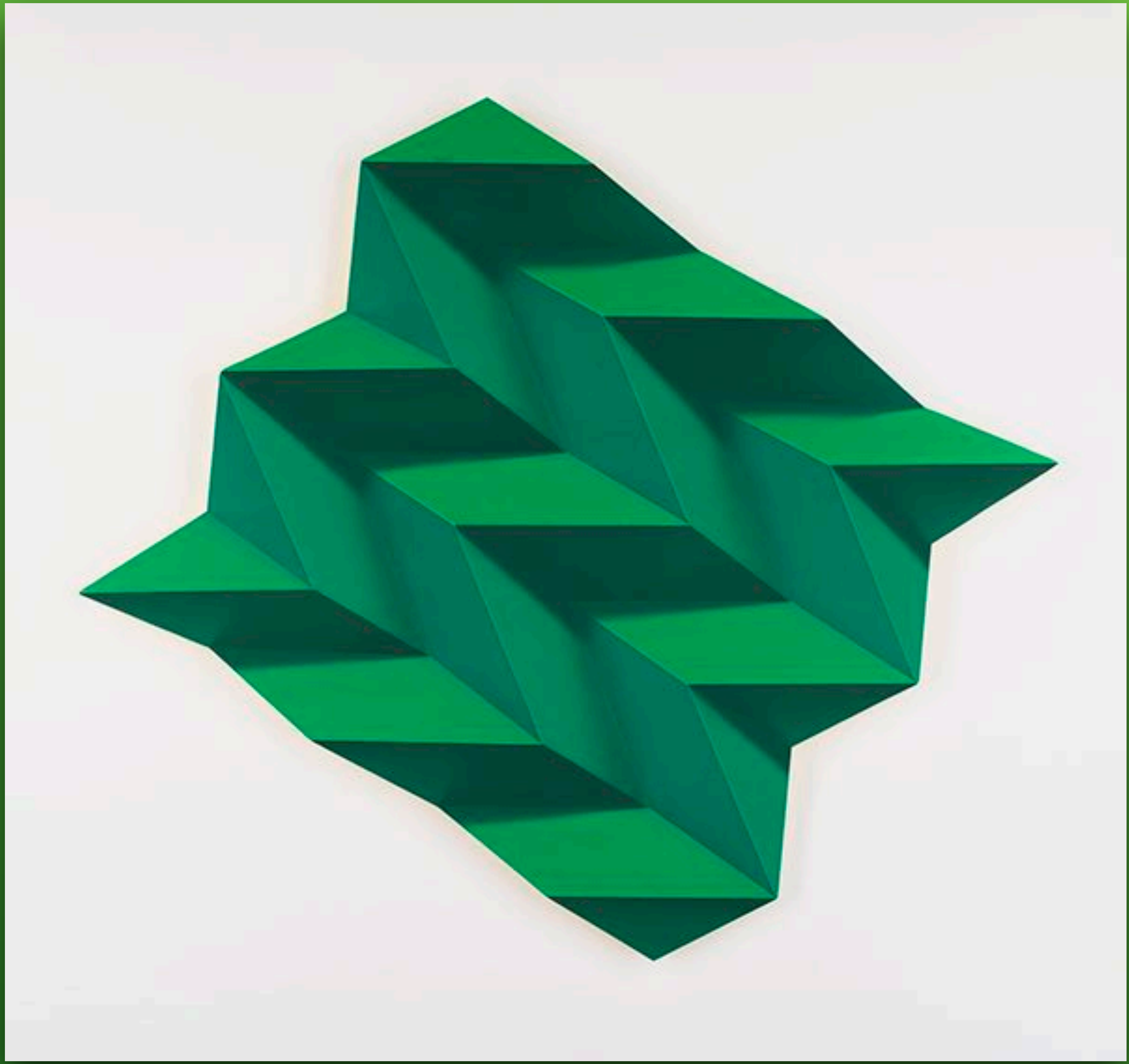
dx^2, dx^5 : not on D3



☆ a probe D3 brane in (x^0, x^1, x^3, x^4)

→4d susy gauge theory

w/ two-dim. defects



3. Supersymmetric D-brane actions

3.0 Simple method (for abelian case)

- **Take the static embedding (simple).**
- **Just throw the bgr data into the formula of DBI actions in**

[Martucci, Rosseel, Van den Bleeken and Van Proeyen '05]

$$S_{Dp}^{(B)} = -\int d^{p+1}x\, e^{-\Phi}\sqrt{-\det(g+\mathcal{F})} + \int \sum_n P[C_n]e^{\mathcal{F}}$$

$$S_{Dp}^{(F)} = \frac{1}{2}\int d^{p+1}x\, e^{-\Phi}\sqrt{-\det(g+\mathcal{F})}\,\bar{\Psi}(1-\Gamma_{Dp})(\Gamma^{\alpha}D_{\alpha} - \Delta + L_{D_p})\Psi$$

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$$S_{Dp}^{(B)} = -\int d^{p+1}x\, e^{-\Phi}\sqrt{-\det(g+\mathcal{F})} + \int \sum_n P[C_n] e^{\mathcal{F}}$$

In particular, at $\mathcal{O}(\varepsilon)$ only the CS-term ↑ contributes.

Leading correction of deform.

D3: $\int P[C_4]$ **D1:** $\int P[C_2]$

D5: $\int P[C_4] \wedge F_2$ **D0:** $\int P[C_1]$

2.4.4 Warm-up: D0-brane in x^0 (~1d defect)

Let us start from a D0-brane embedded in

$$ds_{10}^2 = \Delta\left\{-(dx^0)^2 + (dx^1)^2\right\} + \sum_{I,J=2}^{9}\left(\Delta\delta_{IJ} - \frac{U_I U_J}{\Delta}\right) dx^I dx^J$$

$$\Phi = \frac{3}{2}\ln\Delta\,, \quad C_1 = \frac{1}{\Delta^2}U_J dx^J = \frac{i}{4\Delta^2}\sum_{m=1}^{4}\varepsilon_m(z^m d\bar{z}^m - \bar{z}^m dz^m)$$

$$z^m = x^{2m} + i x^{2m+1}$$

2.4.4 Warm-up: D0-brane in x^0 (~1d defect)

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$$z^m = x^{2m} + ix^{2m+1}$$

The action for bosons and fermions can be written as

$$S_{D0}^{B+F} = \int dx^0 - \frac{1}{2}\partial_0 X^1 \partial^0 X^1 - \frac{1}{2}\sum_{m=1}^{4}(\partial_0 Z^m + i\varepsilon_m Z^m)(\partial^0 \bar{Z}^m - i\varepsilon_m \bar{Z}^m)$$

$$+ \frac{i}{2}\bar{\Psi}\Gamma^0\left(\partial_0 + i\sum_{m=1}^{4}\varepsilon_m \Gamma^{2m(2m+1)}\right)\Psi + \mathcal{O}(\varepsilon^2)$$

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$$+ \frac{i}{2}\bar{\Psi}\Gamma^0\left(\partial_0 + i\sum_{m=1}^{4}\varepsilon_m \Gamma^{2m(2m+1)}\right)\Psi + \mathcal{O}(\varepsilon^2)$$

☆ **The** ▬ **part can be interpreted as a connection, obtained by gauging the U(1) symm. for** Z^m **:**

$$\mathcal{A}_\mu = \delta_\mu^0$$

This looks like a time-like Wilson line with a constant vev ($i\varepsilon_m$ **).**

3.2.1 Result: susy D5 brane action (abelian)

At $\mathcal{O}(\varepsilon)$, the CS-term for the D5 is given by

$I, J = 6, 7, 8, 9$

$$\int P[C_4] \wedge F_2 = \int d^6x - \mathcal{A}_\mu \omega_{IJ} X^J \partial^\mu X^I + \mathcal{O}(\varepsilon^2)$$

$\mu = \{\alpha, a\} =, 0, \cdots, 5$

where

$$\mathcal{A}_\mu = \frac{1}{2} \Xi_{\mu\nu\rho} F^{\nu\rho}$$

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$\mu = \{\alpha, a\} =, 0, \cdots, 5$

where

$$\mathcal{A}_\mu = \frac{1}{2} \Xi_{\mu\nu\rho} F^{\nu\rho}$$

This motivates us to define the twisted covariant derivative for the scalars

$$\mathcal{D}_\mu X^I = \partial_\mu X^I + \mathcal{A}_\mu \omega_{IJ} X^J$$

Then the boson part

$$S^{(B)}_{D5} = \int d^6x - \frac{1}{2}\partial_\mu X^I \partial^\mu X^I - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathcal{A}_\mu \omega_{IJ} X^J \partial^\mu X^I$$

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At $\mathcal{O}(\varepsilon)$, the CS-term for the D5 is given by

$I, J = 6, 7, 8, 9$

$$\int P[C_4] \wedge F_2 = \int d^6x - \mathcal{A}_\mu \omega_{IJ} X^J \partial^\mu X^I + \mathcal{O}(\varepsilon^2)$$

$\mu = \{\alpha, a\} =, 0, \cdots, 5$

where

$$\mathcal{A}_\mu = \frac{1}{2} \Xi_{\mu\nu\rho} F^{\nu\rho}$$

This motivates us to define the twisted covariant derivative for the scalars

$$\mathcal{D}_\mu X^I = \partial_\mu X^I + \mathcal{A}_\mu \omega_{IJ} X^J$$

Then the boson part can be approximately written as

$$S_{D5}^{(B)} = \int d^6x - \frac{1}{2} \mathcal{D}_\mu X^I \mathcal{D}^\mu X^I - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(\varepsilon^2)$$

3.2.1 Result: susy D5 brane action (abelian)

At $\mathcal{O}(\varepsilon)$, in fact, the fermonic part is computed as

Yukawa-like

$$S_{D5}^{(F)} = \int d^6x \, \frac{i}{2} \bar{\Psi} \left[\Gamma^\mu \mathcal{D}_\mu + \frac{1}{4} \Xi_{\mu\nu\rho} \Gamma^{\nu\rho} \omega_{IJ} \partial^\mu X^I \Gamma^J \right] \Psi + \mathcal{O}(\varepsilon^2)$$

where the twisted covariant derivative acts on the spinor as

$$\mathcal{D}_\mu \Psi = \partial_\mu \Psi + \mathcal{A}_\mu \frac{1}{4} \omega_{IJ} \Gamma^{IJ} \Psi$$

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(proven only for the boson.)

$\mathcal{O}(\varepsilon)$

★ **Without approximation for ε, the twisted covariant derivative is exactly at the first order !**

(no higher-order correction!)

3.2.1 Result: susy D5 brane action (abelian)

$$S_{D5}^{(B)} = \int d^6x \sqrt{-g} \left[-\frac{1}{2} g_{\mu\nu} g_{IJ} \mathcal{D}^\mu X^I \mathcal{D}^\nu X^J - \frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F^{\rho\sigma} \right]$$

$$\mathcal{D}_\mu X^I = \partial_\mu X^I + \mathcal{A}_\mu \omega_{IJ} X^J$$

$$g_{\alpha\beta} = \Delta \eta_{\alpha\beta} \,, \;\; g_{ab} = \Delta^{-1} \delta_{ab} \,,$$
$$g_{IJ} = \Delta \delta_{IJ} - \Delta^{-1} U_I U_J$$

* $\sqrt{-g} = 1$, **though.**

(proven only for the boson.)

$\mathcal{O}(\varepsilon)$

★ Without approximation for ε , the twisted covariant derivative is exactly at the first order !

(no higher-order correction!)

3.2.1 Result: susy D5 brane action (abelian)

At $\mathcal{O}(\varepsilon)$, the sum of

$$S_{D5}^{(F)} = \int d^6x \, \frac{i}{2} \bar{\Psi} \left[\Gamma^\mu \mathcal{D}_\mu + \frac{1}{4} \Xi_{\mu\nu\rho} \Gamma^{\nu\rho} \omega_{IJ} \partial^\mu X^I \Gamma^J \right] \Psi + \mathcal{O}(\varepsilon^2)$$

$$S_{D5}^{(B)} = \int d^6x - \frac{1}{2} \mathcal{D}_\mu X^I \mathcal{D}^\mu X^I - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(\varepsilon^2)$$

3.2.1 Result: susy D5 brane action (abelian)

At $\mathcal{O}(\varepsilon)$, the sum of

$$S_{D5}^{(F)} = \int d^6x \, \frac{i}{2} \bar{\Psi} \left[\Gamma^\mu \mathcal{D}_\mu + \frac{1}{4} \Xi_{\mu\nu\rho} \Gamma^{\nu\rho} \omega_{IJ} \partial^\mu X^I \Gamma^J \right] \Psi + \mathcal{O}(\varepsilon^2)$$

$$S_{D5}^{(B)} = \int d^6x - \frac{1}{2} \mathcal{D}_\mu X^I \mathcal{D}^\mu X^I - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(\varepsilon^2)$$

is invariant under

$$\delta_\epsilon X^I = i \bar{\epsilon}(X) \Gamma^I \Psi$$
$$\delta_\epsilon A_\mu = i \bar{\epsilon}(X) \Gamma_\mu \Psi$$
$$\delta_\epsilon \Psi = \Gamma^\mu \Gamma^I \partial_\mu X^I \epsilon(X) + \frac{1}{2} F_{\mu\nu} \Gamma^{\mu\nu} \epsilon(X)$$

(standard)

with a non-constant Killing spinors

$$\epsilon(X) = \epsilon_0 + \frac{1}{4!} \Xi_{\mu\nu\rho} \Gamma^{\mu\nu\rho} X^J \omega_{JK} \Gamma^K \epsilon_0$$

3.2.1 Result: susy D5 brane action (abelian)

At $\mathcal{O}(\varepsilon)$, the sum of

$$S_{D5}^{(F)} = \int d^6x \, \frac{i}{2}\bar{\Psi}\left[\Gamma^\mu \mathcal{D}_\mu + \frac{1}{4}\Xi_{\mu\nu\rho}\Gamma^{\nu\rho}\omega_{IJ}\partial^\mu X^I \Gamma^J\right]\Psi + \mathcal{O}(\varepsilon^2)$$

$$S_{D5}^{(B)} = \int d^6x \, -\frac{1}{2}\mathcal{D}_\mu X^I \mathcal{D}^\mu X^I - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{O}(\varepsilon^2)$$

is invariant under

$$\delta_\epsilon X^I = i\bar{\epsilon}(X)\Gamma^I\Psi$$

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$$\delta_\epsilon \Psi = \Gamma^\mu\Gamma^I\partial_\mu X^I \epsilon(X) + \frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu}\epsilon(X)$$

with a non-constant Killing spinors

$$\epsilon(X) = \epsilon_0 + \frac{1}{4!}\Xi_{\mu\nu\rho}\Gamma^{\mu\nu\rho}X^J\omega_{JK}\Gamma^K\epsilon_0$$

☆ Ask the 10d gravitino variation for this Killing spinor.

☆ the constant part is subject to the projection: $\omega_{IJ}\Gamma^{IJ}\epsilon_0 = 0$

3.2.2 Result: susy D5 brane action (non-abelian)

Non-abelian case: [Myers '99]

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$$S_{D5}^{(B+F)} = \mathrm{STr}\int d^6x - \frac{1}{2}D_\mu X^I D^\mu X^I - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+ \frac{i}{2}\bar{\Psi}\Gamma^\mu D_\mu \Psi - \frac{1}{2}\bar{\Psi}\Gamma^I\left[X^I, \Psi\right] + \frac{1}{4}\left[X^I, X^J\right]^2$$
$$- \mathcal{A}_\mu \omega_{IJ} X^J D^\mu X^I + \frac{i}{8}\bar{\Psi}\mathcal{A}_\mu \omega_{IJ}\Gamma^{IJ}\Psi$$
$$+ \frac{1}{8}\bar{\Psi}D^\mu X^I \Gamma^J \omega_{IJ}\Xi_{\mu\nu\rho}\Gamma^{\nu\rho}\Psi$$
$$- \frac{1}{2}\bar{\Psi}\Xi_{\mu\nu\rho}\Gamma^{\mu\nu\rho}\omega_{IJ}\left[X^I, X^J\right]\Psi$$

$$\mathcal{A}_\mu = \frac{1}{2}\Xi_{\mu\nu\rho}F^{\nu\rho}$$

$$D_\mu \cdot = \partial_\mu - i[A_\mu, \cdot]$$

3.2.2 Result: susy D5 brane action (non-abelian)

Non-abelian case: [Myers '99]

$$S_{D5}^{(B+F)} = \mathrm{STr}\int d^6x - \frac{1}{2}D_\mu X^I D^\mu X^I - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\Psi}\Gamma^\mu D_\mu \Psi - \frac{1}{2}\bar{\Psi}\Gamma^I \left[X^I, \Psi\right] + \frac{1}{4}\left[X^I, X^J\right]^2$$

standard

$$- \mathcal{A}_\mu \omega_{IJ} X^J D^\mu X^I + \frac{i}{8}\bar{\Psi}\mathcal{A}_\mu \omega_{IJ}\Gamma^{IJ}\Psi + \frac{1}{8}\bar{\Psi}D^\mu X^I \Gamma^J \omega_{IJ}\Xi_{\mu\nu\rho}\Gamma^{\nu\rho}\Psi$$

also found in the abelian case

$\partial_\mu \to D_\mu$

$$- \frac{1}{2}\bar{\Psi}\Xi_{\mu\nu\rho}\Gamma^{\mu\nu\rho}\omega_{IJ}\left[X^I, X^J\right]\Psi$$

vanishing in the abelian limit 😡

$$\mathcal{A}_\mu = \frac{1}{2}\Xi_{\mu\nu\rho}F^{\nu\rho}$$

$$D_\mu \cdot = \partial_\mu - i[A_\mu, \cdot]$$

3.2.3 Result: susy D5 brane action (non-abelian)

For a stack of branes, the pull-back of Killing spinors on the brane world-volume is not clear. How?

Recall the abelian case: using a non-constant $\epsilon(X)$, the susy transformations are given by

$$\delta_\epsilon X^I = i\bar{\epsilon}(X)\Gamma^I\Psi$$

$$\delta_\epsilon A_\mu = i\bar{\epsilon}(X)\Gamma_\mu\Psi$$

$$\delta_\epsilon\Psi = \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon(X) + \frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu}\epsilon(X)$$

3.2.3 Result: susy D5 brane action (non-abelian)

For a stack of branes, the pull-back of Killing spinors on the brane world-volume is not clear. How?

Also, recall that the undeformed transformations in the non-abelian case have vanishing terms in the abelian limit:

$$\delta_\epsilon X^I_a = i\bar{\epsilon}\Gamma^I\Psi_a$$

$$\delta_\epsilon A_{\mu a} = i\bar{\epsilon}\Gamma_\mu\Psi_a$$

$$\delta_\epsilon \Psi_a = \Gamma^\mu\Gamma^I D_\mu X^I_a\epsilon + \frac{1}{2}F_{\mu\nu a}\Gamma^{\mu\nu}\epsilon$$

$$-\frac{i}{2}\Gamma_{IJ}[X^I, X^J]_a\epsilon$$

3.2.3 Result: susy D5 brane action (non-abelian)

For a stack of branes, the pull-back of Killing spinors on the brane world-volume is not clear. How?

The non-constant Killing spinor was obtained by the 10d gravitino variation.

(no color index!) but here, the natural extension is to add gauge indices to $\epsilon(X)$ **!**

$$\delta_\epsilon X^I_a = i\bar{\epsilon}^b{}_a(X)\Gamma^I\Psi_b$$

$$\delta_\epsilon A_{\mu a} = i\bar{\epsilon}^b{}_a(X)\Gamma_\mu\Psi_b$$

$$\delta_\epsilon \Psi_a = \Gamma^\mu\Gamma^I D_\mu X^I_b \epsilon^b{}_a(X) + \frac{1}{2}F_{\mu\nu b}\Gamma^{\mu\nu}\epsilon^b{}_a(X) - \frac{i}{2}\Gamma_{IJ}[X^I, X^J]_b\epsilon^b{}_a(X)$$

3.2.3 Result: susy D5 brane action (non-abelian)

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The non-constant Killing spinor was obtained by the 10d gravitino variation.

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with

$$\epsilon^a_b(X) = \delta^a{}_b\epsilon + \frac{1}{4!}\Xi_{\mu\nu\rho}\Gamma^{\mu\nu\rho}X^J_c\omega_{JK}\Gamma^K d^{ac}{}_b\epsilon$$ **and** $$d^{abc} = \frac{1}{2}\mathrm{Tr}(\{T^a, T^b\}T^c)$$
$$\omega_{IJ}\Gamma^{IJ}\epsilon = 0$$

✔

3.3.1 Result: susy D3 brane action (abelian)

Dimensionally reduce $S_{D5}^{(B+F)}$ **:** (**embedded in** $(x^\alpha, x^a) = (x^0, x^1, x^3, x^4)$)

$$S_{D3}^{(B+F)} = \int d^4x - \frac{1}{2}\partial_\mu X^I \partial^\mu X^I - \frac{1}{2}\partial^\mu X^j \partial_\mu X^j - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$- \mathcal{A}_\mu \omega_{IJ} X^J \partial^\mu X^I$$
$$+ \frac{i}{2}\bar{\Psi}\left[\Gamma^\mu \partial_\mu + \mathcal{A}_\mu \frac{1}{4}\omega_{IJ}\Gamma^{IJ} + \frac{1}{4}\Xi_{\mu\nu\rho}\Gamma^{\nu\rho}\omega_{IJ}\partial^\mu X^I \Gamma^J\right]\Psi$$

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$$+ \frac{i}{2}\bar{\Psi}\left[\Gamma^\mu \partial_\mu + \mathcal{A}_\mu \frac{1}{4}\omega_{IJ}\Gamma^{IJ} + \frac{1}{4}\Xi_{\mu\nu\rho}\Gamma^{\nu\rho}\omega_{IJ}\partial^\mu X^I \Gamma^J\right]\Psi$$

☆ **Here, the 2 scalars X^i act on the other scalars in the twisted covariant derivative:**

$$\mathcal{A}_\mu = \Xi_{\mu\nu j}\partial^\nu X^j = \begin{cases} \frac{1}{2}\epsilon_{\alpha\beta}\partial^\beta X^5 & \mu,\nu = \alpha,\beta = 0,1 \\ \frac{1}{2}\epsilon_{ab}\partial^b X^2 & \mu,\nu = a,b, = 3,4 \end{cases} \qquad i,j = 2,5$$

3.3.1 Result: susy D3 brane action (abelian)

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☆ **Lorentz invariance SO(1,3) is broken to SO(1,1)xSO(2) (= due to two 2d defects).**

3.3.1 Result: susy D3 brane action

Just as before, define the twisted cov. derivative:

$$S_{D3}^{(B+F)} = \int d^4x - \frac{1}{2}\mathcal{D}_\mu X^I \mathcal{D}^\mu X^I - \frac{1}{2}\partial^\mu X^j \partial_\mu X^j - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+ \frac{i}{2}\bar{\Psi}\left[\Gamma^\mu \mathcal{D}_\mu + \frac{1}{4}\Xi_{\mu\nu\rho}\Gamma^{\nu\rho}\omega_{IJ}\partial^\mu X^I \Gamma^J\right]\Psi + \mathcal{O}(\varepsilon^2)$$

with

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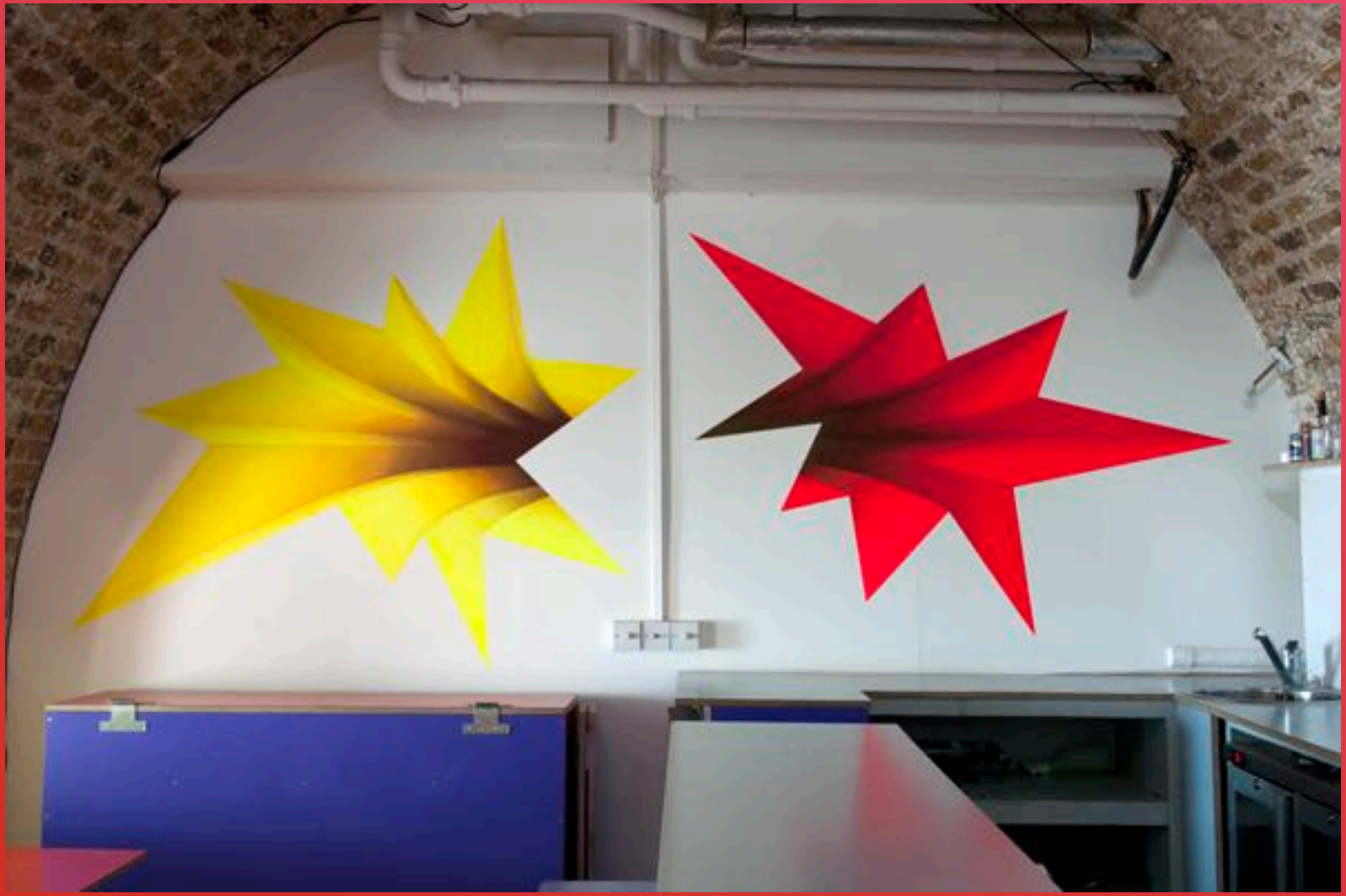
☆ **The effect of deformation was absorbed into the connections and 2 scalars X^i look undeformed.**

☆ **Again, for the boson, it was proven that the covariant derivative includes only the first order $\mathcal{O}(\varepsilon)$.**

☆ **Susy transformations are also obtained by dimensional reduction. The Killing spinor is subject to the same projection: $\omega_{IJ}\Gamma^{IJ}\epsilon = 0$, 8 conserved charges.**

(non-abelian action is omitted.)

4. Thoughts



at royal academy of arts, London

Defects...?

4.1 Thoughts

☆ **We have considered from a 10d (geometrical) perspective**

- **D5 theory as 6d undeformed theory coupled to two 3d defects**
- **D3 theory as 4d undeformed theory coupled to two 2d defects.**

Can we rewrite the leading correction (the CS-term) in a more reasonable form?

4.1 Thoughts

☆ **We have considered from a 10d (geometrical) perspective**

- **D5 theory as 6d undeformed theory coupled to two 3d defects**
- **D3 theory as 4d undeformed theory coupled to two 2d defects.**

Can we rewrite the leading correction (the CS-term) in a more reasonable form?

Not quite: for the D3-brane case,

$$\int P[C_4] \sim \int \mathcal{B}_D^{(2)IJ} \wedge \mathcal{J}_D^{(2)IJ}$$

$$\mathcal{J}_D^{(2)IJ} = d\mathcal{J}_R^{(1)IJ}$$

could be seen as deforming the Lagrangian

$$\mathcal{L}_{\text{undeformed}} \to \mathcal{L}_{\text{undeformed}} + \mathcal{B}_D^{(2)IJ} \wedge \mathcal{J}_D^{(2)IJ}$$

but this form does not yield a term in the form of (3-form x 3-form) for D5.

4.2 That's why…

~ **Extended Gauge Theory Deformations**
from
Flux Backgrounds ~

4.3 Not….

~ **Higher-dimensional Defects** ~

from

Flux Backgrounds

5. Summary / Outlook

5.1 Summary

- **Constructed R-R flux backgrounds using Ω-deform.**

- **Focused on the microscopic descriptions of supersymmetric gauge theories realized from various branes: they turned out to have the structure of twisted covariant derivative.**

At the full orders: Boson: OK, but Fermion: not clear. Also, higher derivatives?

D5: connection containing the field strength. Relation to [Ganor, '17] ?

- **The effect of deformations:**
  - D0 → a line defect
  - D3 "→" 2d defects
  - D5 "→" 3d defects …

→Extracting (2d/3d) surface operators from the Lagrangian is not clear: How?

- **How do we connect with the literatures of susy gauge theories?**

: missing, idk

Thank you!