Pure glue in $6d$ and $4d$ quivers

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Curious observation

- Consider $\mathcal{N}=1$ $SU(3)$ SQCD with nine flavors
- Naively, this is IR free as $N_f = 3N$
- However, $SU(3)$ is special as baryons are marginal

$$W = QQQ + \widetilde{Q}\widetilde{Q}\widetilde{Q}$$

- Have a seven dimensional conformal manifold passing through zero coupling (Leigh, Strassler; Green, Komargodski, Seiberg, Tachikawa, Wecht)
- On general point of the conformal manifold all symmetry broken

John Schwarz, lessons, Strings 2018
(50th Anniversary of string theory)

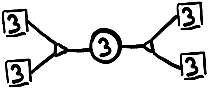
Lesson #2: Take "coincidences" seriously.

Example 1: The massless states of type IIA superstring theory correspond to the massless states of 11d supergravity on a circle. This was known for more than a decade before it was taken seriously.

Example 2: It was well known that the Lorentzian conformal group in d dimensions is the same as the Anti de Sitter isometry group in $d+1$ dimensions many years before AdS/CFT duality was proposed.

Question from Zohar

- Such $\mathcal{N}=1$ conformal manifolds with single gauge group are rare
- With $\mathcal{N}>1$ typically have duality groups acting on the manifold
- The theory can be thought as $SU(3)$ gauging of two tri-fundamentals of $SU(3)$ (9 fundamental flavors is $3\times 3\times 3$)
- This is very reminiscent of class $\mathcal{S}$



- I ; What is the duality group?
- II ; Is there a geometric picture?

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Lesson #3: When working on hard problems explore generalizations with additional parameters.

This lesson seems to be widely appreciated. There are many examples in the literature.

A couple of well-known examples are the Ω background for $\mathcal{N}=2$ gauge theories and the $\mathbb{Z}_k$ orbifold generalization of $AdS_4 \times S^7$, which plays an important role in ABJM theory.

Plan

- In recent years we had learnt quite a bit about relations between $\mathcal{N} = 1$ theories in four dimensions and compactifications from six dimensions

- The understandings allow a rather algorithmic way to answer such questions

- We will today then use that to answer Zohar's questions

- We will engineer systematically the cute model at hand as compactification of $6D$ CFT and put it in a large class of theories

- This will imply various interesting properties, such as duality, that the theories of this class enjoy. We will also discuss generalities of compactifications of simple $6d$ theories to $4d$.

Relations between six and four dimensions

- We can engineer many (all?) supersymmetric conformal field theories in $4d$ starting from a $6d$ SCFT.

  1. Class $\mathcal{S}$ – $(2,0)$ theory on surfaces with particular flux for R symmetry (Gaiotto 2008)
  2. $\mathcal{N}=1$ class $\mathcal{S}$ – $(2,0)$ with more general flux (Benini, Tachikawa, Wecht - 2009, BBBW 2012)
  3. More general $(1,0)$ theories with arbitrary flux, for example ADE conformal matter (Gaiotto, SSR; Ohmori, Shimizu, Tachikawa, Yonekura; Kim, SSR, Vafa, Zafrir)

- For most no Lagrangian description is known in four dimensions
- For some $((A_0, A_1), (A_0, A_2), (A_1, A_1), (D_4, A_0)$ conformal matter) Lagrangians are known (Gaiotto; Gadde, SSR,Willett; Gaiotto, SSR and SSR, Vafa, Zafrir; Kim, SSR,Vafa,Zafrir)
- For some compactifications more is known (tori, AD theories (Maruyoshi, Song))
- Work on predicting properties of $4d$ theories (Heckman and colaborators)

Choices: 6d

- Starting from six dimensions we can derive algorithmically some predictions about four dimensional theories

- The $4d$ theories are labeled by choices: $6d$ theory $\mathcal{T}$, surface $\mathcal{C}$, flux $\mathcal{F}$

- Can predict symmetries $G^{4d}(\mathcal{T},\mathcal{F})$, anomalies,

$$I_{4d} = \int_{\mathcal{C}} \mathcal{I}_{6d}(\mathcal{T},\mathcal{F}),$$

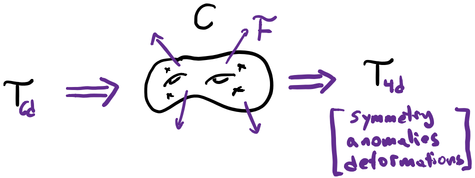
- and can predict the marginal and relevant operators in general

- Typically expect to have $3g-3+s$ exactly marginal deformations related to complex structure moduli. We can have more marginal deformations related to holonomies for symmetries

Punctures: 5d

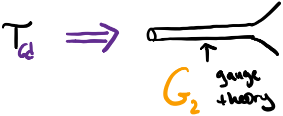
- Going down to $4d$ it is useful to make a stop in $5d$
- To understand punctured surfaces we can first compactify on a circle to get an effective $5d$ description and study boundary conditions
- In some cases the theory in $5d$ is a gauge theory ($(2,0)$ to $\mathcal{N}=2$ SYM)
- In those cases natural (maximal) boundary conditions will turn turn gauge degrees of freedom to flavor
- Flavor symmetry associated to every (maximal) puncture being the $5d$ gauge symmetry
- Can compute anomalies of this symmetry

Road to 4d: summary



$$G = G_1[T_{6d}, F] \times G_2[T_{6d}, C]$$

G_2 : 5d gauge symmetry per puncture



Reverse Engineering the algorithm

- Let us now follow the same algorithm starting from $4d$ and trying to understand what is the geometric picture could result in our model
- The starting point is the $SU(3)$ SQCD with nine flavors
- We will learn that in this particular case the story is so constrained that the answer, assuming it exists, is evident

The theories: $4d$

- What are the essential bits we can take from the theory?
- First, we follow the original intuition and assume that the model is obtained by gluing two blocks, the trifundamentals of $SU(3)$, together by gauging an $SU(3)$ symmetry
- This implies that if a geometric picture exists then we should have (maximal) punctures with $SU(3)$ symmetry
- If we combine many trifundamentals together by gauging $SU(3)$ symmetries in general we will not get any global symmetry, implying that the $6d$ model should not have any continuous symmetry except the R symmetry

Implications about punctures: $5d$

- The fact that the symmetry of the punctures is $SU(3)$ implies that if we take the putative $6d$ theory on a circle it should have an effective description as $SU(3)$ gauge theory

- The fact that the theory in $6d$ should have no symmetry implies that the effective theory should have no matter

- We are after pure $SU(3)$ gauge theory in five dimensions!!

- The cubic 't Hooft anomalies of the puncture symmetry and mixed anomaly of puncture symmetry and R symmetry imply that the theoryhas CS term with level nine

- Interestingly that is the precisely the level for which pure $SU(3)$ theory is conjectured to have a UV completion as a $6d$ SCFT (Jefferson, Kim, Vafa, Zafrir)

Implications about the uplift: $6d$

- Level nine $SU(3)$ gauge theory in $5d$ has a UV completion as a twisted compactification of pure glue $SU(3)$ $(1,0)$ gauge theory

- Pure glue $SU(3)$ theory (with a tensor) is an anomaly free gauge theory (Seiberg; Bershadsky, Vafa)

- It has $\mathbb{Z}_2$ symmetry, complex conjugation, and no continuous global symmetry

- It is not known what will be an effective description in five dimensions of a compactification of this model. Twisting with the $\mathbb{Z}_2$ it is conjectured that the description is level nine $SU(3)$ gauge theory

- Have a conjecture for the $6d$ origin of our model!! Apply the algorithm and figure out general compactifications

Interlude: Pure glue in $6d$

- There is very limited number of non anomalous pure gauge $6d$ theories: $SU(3)$, $SO(8)$, F_4, E_6, E_7, and E_8
- $(TrF^2)^2$ anomaly canceled by Green-Schwarz mechanism, TrF^4 anomaly is there in pure gauge theories in general, $SU(2)$ and G_2 ruled out by Witten anomaly
- Can compute the eight form anomaly polynomial for these models
- Integrating over Riemann surface gives predictions for $4d$,

$$a = \frac{3}{16}(\frac{12}{\lambda_G} - d_G + 1)(g-1), \quad c = \frac{1}{8}(\frac{33}{2\lambda_G} - d_G + 1)(g-1)$$

G	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_8
d_G	8	28	52	78	133	248
λ_G	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{5}{108}$	$\frac{1}{32}$	$\frac{1}{54}$	$\frac{1}{100}$

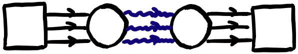
Checks: anomalies

- In case of twisted compactifications of $SU(3)$ we can add punctures,

$$a = \frac{3}{32}(82(g-1)+33s)\,, \quad c = \frac{1}{16}(118(g-1)+51s)$$

- What are the theories reproducing these anomalies?

- Claim: the following theory corresponds to four puncture sphere

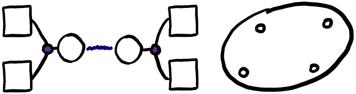


- Theory built from three trifundamentals with $SU(3)$ gauging and turning on baryonic superpotential

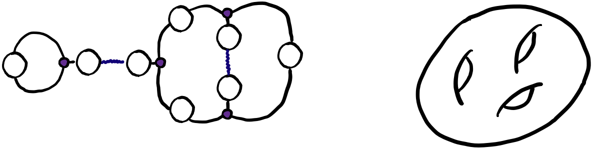
General surfaces

- Building blocks

- Four punctured sphere



- General surface



- All anomalies match

Twists

- The fact that we know the $5d$ description only with twisted compactification implies that the punctures we have come with twist lines around them
- The twist is $\mathbb{Z}_2$ which implies that we cannot have odd number of punctures on a sphere
- In particular we do not have a three punctured sphere
- Such an object might exist but some of the punctures have to be not twisted and we do not know what these would be

Four punctured sphere: conformal manifold

- The expected conformal manifold of the four punctured sphere, preserving the puncture symmetry, is one dimensional, corresponds to complex structure

- Claim: the one dimensional locus corresponding to complex structure modulus does not pass through zero coupling

- This is consistent with no three punctured sphere: the decomposition to three punctured spheres gives strongly coupled model

Checks: conformal manifold

- There are various additional checks we can perform
- duality: the supersymmetric partition functions are invariant under exchanges of punctures (checked in perturbative expansion)
- The index to low orders is given,

$$1+\left(3g-3+s+\sum_{j=1}^{3}(\mathbf{10}_j-\mathbf{8}_j)\right)q\,p+\cdots$$

- Conformal manifold of dimension $3g-3+s$, no relevant operators, each puncture has ten marginal operators two of which are exactly marginal

Checks: closing punctures

- We can close punctures by giving vevs to operators charged under puncture symmetry

- Natural operators are the marginal in **10** of $SU(3)$, the vev breaks completely the symmetry

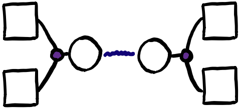
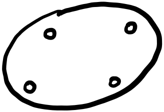
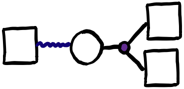
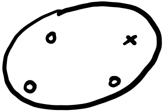
- However, we obtain a puncture with no symmetry, because of the twist puncture cannot be removed, we call the new kind of punctures empty punctures

- Index, $1 + (3g - 3 + s + s' + \sum_{j=1}^{3}(\mathbf{10}_j - \mathbf{8}_j))\, q\, p + \cdots$, maximal punctures behave as triplets of empty punctures

- On the conformal manifold of a theory with empty punctures there are subloci where triplets of empty punctures collide to form maximal punctures and symmetry is enhanced

Answer to Zohar's question

- We can now answer Zohar's question
- The $SU(3)$ SYM with nine flavors is obtained by taking the four punctured sphere and closing one maximal puncture
- The resulting theory is a sphere with three maximal punctures and one empty puncture, which sits on the conformal manifold of sphere with ten empty punctures
- The conformal manifold is seven dimensional matching the field theory analysis and we expect the mapping class group of sphere with ten punctures to act on the manifold

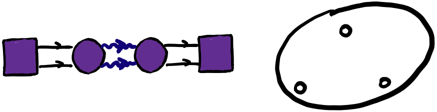
Generalization to $SO(8)$ glue: $6d$ and $5d$

- We can try to generalize to other pure glue theories
- For that useful to have a gauge theory description in $5d$
- Such a description is conjectured for $SO(8)$ twisted by the triality ($\mathbb{Z}_3$) to be pure $SU(4)$ gauge theory in $5d$ with Chern-Simons term at level eight. For other pure glue no such description is known at the moment
- This gives predictions for anomalies and puncture symmetries
- Can we find the four dimensional models?

Generalization to $SO(8)$ glue: $4d$

- The anomalies predicted from $6d$ and $5d$ for the punctures are reproduced by an octet of fundamental fields of $SU(4)$ with R charge half

- The basic building block is a pair of bifundamental fields of $SU(4)$ with the trinion given here



- Trinion is fine as the twist is $\mathbb{Z}_3$

- Have $SU(4)^2 \times SU(2)^2$ symmetry and the claim is that it enhances to $SU(4)^3$ at a locus on conformal manifold

- Building arbitrary theories match $6d$ predictions

Comments

- We have obtained a large class of $4d$ theories by compactifying $SU(3)$ and $SO(8)$ pure glue theory in $6d$ to $4d$
- The former construction is completely Lagrangian and is one of the simplest to date
- The latter case involves gauging symmetries appearing only at some loci of the conformal manifold, a feature shared by other compactifications (see "Lagrangians" for (A_0, A_2), (A_1, A_1), (D_4, A_0))
- Possible to reverse the reduction because had no symmetry, symmetry complicates logic as it adds many things to play with (holonomies, deformations)

Open questions

- Can we understand compactifications of other pure glue gauge theories?
- Can we understand compactifications/punctures with no twists?
- How precisely the duality acts on the conformal manifold?
- Are there other cute $4d$ questions which can be answered using the uplift to $6d$?

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Lesson #1: If a theory developed for purpose A turns out to be better suited for purpose B, modify your goal accordingly.

The original goal of string theory was a theory of hadrons, but it turned out to work better as a theory of quantum gravity and unification. The massless particles should be identified as gauge particles and a graviton rather than vector mesons and a Pomeron.

Thank you!!