A duality network of linear quivers

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SUSY gauge theories, their dualities and deformations
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$\int dk \prod$ Doktoratskolleg Particles and Interactions

Conjecture:

Supersymmetric QCD is dual to a large number of linear quiver gauge theories.

Based on:

FB, V.P. Spiridonov, 1605.06991

FB, V.P. Spiridonov, to appear

Supersymmetric QCD:

- SUSY gauge theory with gauge group $SU(N_c)$ and $SU(N_f) \times SU(N_f) \times U(1)_B$ flavour symmetry
- $\mathcal{N} = 1$ supersymmetry $\rightarrow$ R-symmetry $U(1)_R$
- Axial $U(1)$ anomalous
- N_f quarks and squarks in chiral multiplets $\mathbf{Q}^i$ and $\tilde{\mathbf{Q}}_i$
- $SU(N_c)$ gauge field part of vector multiplet $\mathbf{V}$

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
$\mathbf{Q}^i$	f	f	1	1	$(N_f - N_c)/N_f$
$\tilde{\mathbf{Q}}_i$	$\bar{f}$	1	$\bar{f}$	-1	$(N_f - N_c)/N_f$
$\mathbf{V}$	adj	1	1	0	1

Gauge invariant operators ("hadrons"):

$$M^i_j = \mathbf{Q}^i \tilde{\mathbf{Q}}_j$$
$$B^{i_1,\ldots,i_{N_c}} = \mathbf{Q}^{i_1} \cdots \mathbf{Q}^{i_{N_c}}$$
$$B_{i_1,\ldots,i_{N_c}} = \tilde{\mathbf{Q}}_{i_1} \cdots \tilde{\mathbf{Q}}_{i_{N_c}}$$

Seiberg, ‘94:

$3N_c/2 < N_f < 3N_c$: $\exists$ IR fixed point $\rightarrow$ SCFT

$\rightarrow$ dual description with gauge group $SU(N_f - N_c)$

eSQCD $\Longleftrightarrow$ mQCD

$\rightarrow$ Seiberg duality

Matter content of mSQCD:

	$SU(\tilde{N}_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
$\mathbf{q}^i$	f	$\bar{f}$	1	$N_c/\tilde{N}_c$	N_c/N_f
$\tilde{\mathbf{q}}_i$	$\bar{f}$	1	f	$-N_c/\tilde{N}_c$	N_c/N_f
$\mathbf{V}$	adj	1	1	0	1
$\mathbf{M}$	1	f	$\bar{f}$	0	$\tilde{N}_c/N_f$

for $\tilde{N}_c = N_f - N_c$

Checks?

- ‘t Hooft anomaly matching (‘t Hooft, ‘79)
  - Chiral anomalies of different descriptions should match
  - Satisfied for $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$ of Seiberg dual theories!
- BPS state counting
  - Superconformal index should be independent of description
  - Also satisfied, but not obvious - hard to prove!

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$\mathcal{N} = 1$ SUSY generated by Q^α, $\tilde{Q}^{\dot{\alpha}}$, four real charges

BPS states preserve only part of the supersymmetry

$\rightarrow$ also known as short multiplets/representations

How to count them?

Kinney, Maldacena, Minwalla, Raju, '05:

Countable in SCFTs by computing the superconformal index (SCI):

$$\mathcal{I} = \mathrm{Tr}(-1)^{\mathcal{F}} e^{-\beta H} p^{\frac{R}{2}+J_R+J_L} q^{\frac{R}{2}+J_R-J_L} \prod_i z_i^{G_i} \prod_j y_j^{F_j}$$

$$\begin{aligned} &R \ldots \text{R-charge}, \qquad \beta \ldots \text{chemical potential} \\ &J_L, J_R \ldots \text{Cartan generators of } SU(2)_L \times SU(2)_R \\ &G_i, F_j \ldots \text{generators of gauge and flavour groups} \\ &p, q, z_i, y_j \ldots \text{complex fugacities} \end{aligned}$$

Contributions only from $H = E - 2J_L - \frac{3}{2}R = 0$

Properties

- Only BPS states contribute
- Not affected by SUSY preserving deformations $\rightarrow$ invariant under RG flow!
- Topological quantity
- Only depends on group-theoretic information

Gauge invariance: rewritten as

$$\mathcal{I}(p,q,y) = \int_G d\mu(g) \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} i(p^n, q^n, y^n, z^n)\right),$$

Single particle index $i(p,q,y,z)$ depends on characters χ_{adj}, χ_f, $\chi_{\bar{f}}$, ...

Information easily available!

In principle easy recipe for checking dualities:

1. Deduce representations and their characters from field content
2. Compute SCI for both theories
3. Indices match if theories actually dual

In practice hard:

- Integrals computable exactly only in rare cases
- Identification of integrals hard to prove

Dolan, Osborn, ‘08:

Solution: rewrite SCIs as elliptic hypergeometric integrals

Many integral identities known and applicable to SCIs!

$\rightarrow$ Fruitful interchange between mathematics and physics

SCI of eSQCD:

$$\mathcal{I}_{\mathrm{eSQCD}} = I_n^{(m)}(\mathbf{s},\mathbf{t}) = \kappa_n \int_{\mathbb{T}^n} \frac{\prod_{j=1}^{N_c}\prod_{l=1}^{N_f}\Gamma(s_l z_j, t_l^{-1} z_j^{-1})}{\prod_{1\le j<k\le N_c}\Gamma(z_j z_k^{-1}, z_j^{-1} z_k)} \prod_{k=1}^{N_c-1}\frac{dz_k}{2\pi i z_k},$$

with $N_c = n+1$, $N_f = m+n+2$ and $\mathbf{s},\mathbf{t}$ contain flavours

$$\Gamma(z) := \Gamma(z;p,q) = \prod_{j,k=0}^{\infty}\frac{1-z^{-1}p^{j+1}q^{k+1}}{1-zp^j q^k}$$

Field content can be read off directly!

	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
$\mathbf{Q}^i$	f	f	1	1	$(N_f - N_c)/N_f$
$\tilde{\mathbf{Q}}_i$	$\bar{f}$	1	$\bar{f}$	-1	$(N_f - N_c)/N_f$
$\mathbf{V}$	adj	1	1	0	1

$$I_n^{(m)}(\mathbf{s},\mathbf{t}) = \int_{\mathbb{T}^{N_c-1}} \frac{\prod_{j=1}^{N_c}\prod_{l=1}^{N_f}\Gamma(s_l z_j, t_l^{-1} z_j^{-1})}{\prod_{1\le j<k\le N_c}\Gamma(z_j z_k^{-1}, z_j^{-1} z_k)} \prod_{k=1}^{N_c-1}\frac{dz_k}{2\pi i z_k}$$

Characters: $\chi_{SU(N),f} = \sum_i^N x_i$, $\chi_{SU(N),\bar{f}} = \sum_i^N x_i^{-1}$,
$\chi_{SU(N),adj} = \sum_{1\le i,j\le N} x_i x_j^{-1} - 1$

	$SU(\tilde{N}_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
$\mathbf{q}$	f	$\bar{f}$	1	$N_c/\tilde{N}_c$	N_c/N_f
$\tilde{\mathbf{q}}$	$\bar{f}$	1	f	$-N_c/\tilde{N}_c$	N_c/N_f
$\mathbf{V}$	adj	1	1	0	1
$\mathbf{M}$	1	f	$\bar{f}$	0	$\tilde{N}_c/N_f$

$$\mathcal{I}_{\text{mSQCD}} = \prod_{j,k=1}^{N_f} \Gamma(t_j s_k) \int_{\mathbb{T}^{\tilde{N}_c-1}} \frac{\prod_{j=1}^{\tilde{N}_c} \prod_{l=1}^{N_f} \Gamma({s'}_l z_j, {t'}_l^{-1} z_j^{-1})}{\prod_{1\leq j<k\leq \tilde{N}_c} \Gamma(z_j z_k^{-1}, z_j^{-1} z_k)} \prod_{k=1}^{\tilde{N}_c-1} \frac{dz_k}{2\pi i z_k}$$

$$= \prod_{j,k=1}^{N_f} \Gamma(t_j s_k) I_m^{(n)}(\mathbf{s}', \mathbf{t}')$$

Rains, ‘03:

$$I_n^{(m)}(\mathbf{s},\mathbf{t}) = \prod_{j,k=1}^{n+m+2} \Gamma(t_j s_k) I_m^{(n)}(\mathbf{s}',\mathbf{t}')$$

Proves matching of SCIs for Seiberg duality, i.e.

$$\mathcal{I}_{\mathrm{eSQCD}} = \mathcal{I}_{\mathrm{mSQCD}}$$

Similar proofs for countless other SCIs

Strategy so far:

1. Identify dual theories (e.g. Seiberg duality)
2. Extract group theoretic informaton (representations and characters)
3. Compute SCIs
4. Use theory of elliptic hypergeometric functions to prove their equivalence

Possible to turn this around?

New Strategy:

1. Derive new integral identities
2. Identify integrals as SCIs
3. Read off field content
4. Conjecture new dualities

Spiridonov, '08, FB, Spiridonov, 1605.06991:

Recursion relation for elliptic hypergeometric integrals on A_n root system:

$$I_n^{(m+1)}(\mathbf{s},\mathbf{t}) = \mathbf{Q}_n^m I_n^{(m)}(\tilde{\mathbf{s}},\mathbf{t})$$

$$\mathbf{Q}_n^m := \zeta(v) \times \int_{\mathbb{T}^n} \frac{\prod_{j=1}^{n+1} \Gamma(\frac{t_{n+m+3} w_j}{v^n}) \prod_{l=1}^{n+2} \Gamma(\frac{s_l}{v w_j})}{\prod_{1\leq j<k\leq n+1} \Gamma(w_j w_k^{-1}, w_j^{-1} w_k)} \prod_{k=1}^{n} \frac{dw_k}{2\pi \mathrm{i} w_k}$$

$$\zeta(v) = \frac{\kappa_n}{\Gamma(v^{n+1})} \prod_{l=1}^{n+2} \frac{\Gamma(t_{n+m+3} s_l)}{\Gamma(v^{-n-1} t_{n+m+3} s_l)}$$

$$\tilde{\mathbf{s}} = (v w_1, \ldots, v w_{n+1}, s_{n+3}, \ldots, s_{n+m+3})$$

L.h.s.: SCI of SQCD, r.h.s: SCI of linear quiver!

Flavour symmetries different from SQCD, subgroups!

$$SU(N_f - N_c - 1) \times SU(N_c + 1) \times U(1) \subset SU(N_f)$$
$$SU(N_f - 1) \times U(1) \subset SU(N_f)$$

Superpotential?

	$SU(N_c)_1$	$SU(N_c)_2$	$SU(N_f-N_c-1)$	$SU(N_c+1)$	$SU(N_f-1)$	$U(1)$	$U(1)$	$U(1)_B$	$U(1)_R$
$\mathbf{A}_1$	1	f	1	1	1	$\frac{1}{N_c}$	$-\frac{N_c-1}{N_c}$	N_c	$\frac{N_f-N_c-(N_c-2)(N_f-N_c-2)}{2N_f}$
$\mathbf{A}_2$	1	$\bar{f}$	1	$\bar{f}$	1	$-\frac{1}{N_c}$	$-\frac{1}{N_c(N_c+1)}$	0	$\frac{1}{N_f}$
$\mathbf{A}_3$	f	1	1	1	f	$-\frac{1}{N_f-1}$	0	1	$\frac{N_f-N_c}{2N_f}$
$\mathbf{A}_4$	$\bar{f}$	f	1	1	1	$\frac{1}{N_c}$	$\frac{1}{N_c}$	-1	$\frac{N_f-N_c-2}{2N_f}$
$\mathbf{A}_5$	$\bar{f}$	1	$\bar{f}$	1	1	0	$-\frac{1}{N_f-N_c-1}$	-1	$\frac{N_f-N_c}{2N_f}$
$\mathbf{M}_1$	1	1	1	1	1	-1	1	N_c	$1+\frac{N_c(N_c+2-N_f)}{2(N_f-1)}$
$\mathbf{M}_2$	1	1	1	$\bar{f}$	1	1	$\frac{1}{N_c+1}$	0	$\frac{N_f-N_c}{N_f}$
$\mathbf{M}_3$	1	1	1	f	1	-1	$-\frac{1}{N_c+1}$	$-N_c$	$\frac{N_c(N_f-N_c)}{2N_f}$

Recursion can be iterated!

Example: SQCD with $N_c = 3$ and $N_f = 6$: $\mathcal{I}_{\text{eSQCD}} = I_2^{(2)}$

Recursion relation leads to

$$
\begin{aligned}
I_2^{(2)} &= \mathbf{Q}_2^1 I_2^{(1)} \\
&= \mathbf{Q}_2^1 \mathbf{Q}_2^0 I_2^{(0)}
\end{aligned}
$$

Two distinct linear quivers!

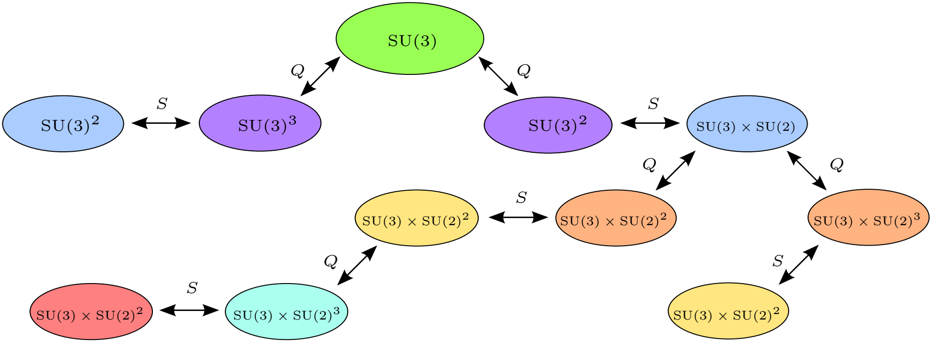
New dualities can be combined with Seiberg duality, schematically:

$$I_n^{(m)} = c_m^n I_m^{(n)} \qquad \mathrm{S}$$
$$I_n^{(m+1)} = \mathbf{Q}_n^m I_n^{(m)} \qquad \mathrm{Q}$$

Leads to a large duality web of linear quivers dual to SQCD!

True both for eSQCD and mSQCD.

Duality network for SQCD with $N_c = 3$ and $N_f = 6$:



Summary:

- SCIs serve as a check for dualities, e.g. Seiberg duality, AdS/CFT, ...
- SCIs can be written in terms of elliptic hypergeometric functions
- SCIs of dual theories can be proven to be identical
- We have used new identities to deduce new dualities and gathered evidence for the statement that

  SQCD is dual to a large number of linear quiver gauge theories.

Outlook:

- Gain better understanding of these dualities
- Find connections to other (linear) quivers
- Embed the dualities in string theory
- Explore possible relations to topological QFT and 2d lattice models
- Find more related dualities (FB, Spiridonov, to appear)