GRAVITY & DUALITY

Bern, July 2016

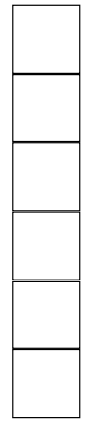
Yang-Mills and Duality

- S-duality of D=4 N=4 SYM
- Classical symmetry of free theory. Quantum symmetry for interacting theory
- D=5 N=4 SYM: strong coupling limit is (2,0) theory
- Free theory: $A_m \longrightarrow B_{mn}$
- Gravity ~ $(YM)^2$. Gravitational analogues?

Gravitational Duality?

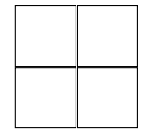
- Dual Gravitons, S-Duality for Linearised Gravity
- Magnetic Mass, NUT charge, KK monopoles
- Non-linear: GR & M-theory
- 6D conformal theories: origin of dualities in 4-D SYM and SUGRA. Highly symmetric
- (2,0): M5 -brane. (4,0): gravitational analogue?
  $A_m \longrightarrow B_{mn}$, $g_{mn} \longrightarrow C_{mnpq}$

Maxwell in D-dimensions

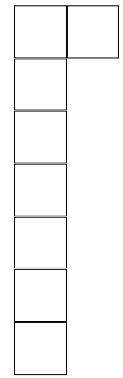
- Photon A_μ 
- Dual photon: n=D-3 form $\tilde{A}_{\mu_1 \ldots \mu_n}$ 
  $F = *\tilde{F}$
- Magnetic charges: D-4 branes.
  A has Dirac strings, or connection on non-trivial bundle, Ã well-defined
- Electric charges: 0-branes.
  Ã has Dirac string singularities, A OK
- YM? No non-abelian theory for Ã

Linearised Gravity

- **Graviton** $h_{\mu\nu}$  $(1,1)$

  Field strength $R_{\mu\nu\rho\sigma}$  $(2,2)$

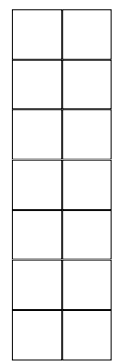
- **Dual Graviton**

  $\tilde{h}_{\mu_1 \ldots \mu_n \nu}$ $(n,1)$ 

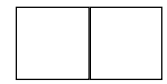
  n=D-3

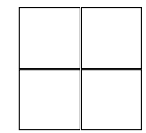
- **Double Dual Graviton**

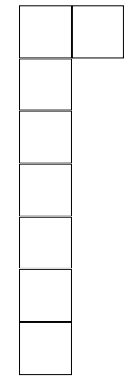
  $\tilde{\tilde{h}}_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}$

  (n,n) 

Linearised Gravity

- **Graviton** $h_{\mu\nu}$  $(1,1)$

  Field strength $R_{\mu\nu\rho\sigma}$  $(2,2)$

- **Dual Graviton** 

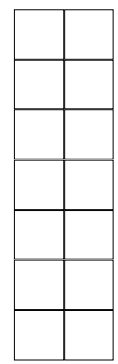
  $\tilde{h}_{\mu_1 \ldots \mu_n \nu}$ $(n,1)$

  $\tilde{R}_{\mu_1 \ldots \mu_{n+1}\, \rho\sigma}$

  $(n+1,2)$

  n=D-3

- **Double Dual Graviton**

  $\tilde{\tilde{h}}_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}$ 

  (n,n)

  $\tilde{\tilde{R}}_{\mu_1 \ldots \mu_{n+1}\, \nu_1 \ldots \nu_{n+1}}$

  $(n+1,n+1)$

Field strengths are Dual:

$$n = D - 3$$

$$R \qquad (2,2)$$

$$\tilde{R} = *R \qquad (n+1,2)$$

$$\tilde{\tilde{R}} = *R* \qquad (n+1,n+1)$$

$$R_{\mu\rho\nu}{}^{\rho} = 0 \qquad \leftrightarrow \qquad \tilde{R}_{[\mu_1 \ldots \mu_n \mu_{n+1} \nu]\rho} = 0$$

$$R_{[\mu\nu\rho]\sigma} = 0 \qquad \leftrightarrow \qquad \tilde{R}_{\mu_1 \ldots \mu_n \rho\, \nu}{}^{\rho} = 0$$

- Hull 2000: Dual graviton, double dual graviton in D dims, motivated by 6-D CFT
- West 2001: Dual graviton & E_{11}
- Bekaert, Boulager & Henneaux 2002: No interactions for dual graviton, no dual formulation of GR
- Non-linear action with both
  West 2001, Boulanger & Hohm 2008
  D=11 Sugra: Bergshoeff, de Roo & Hohm

D=4 S-Dualities

- **Electromagnetic duality**
  SL(2,Z) S-duality of N=4 SYM $\quad A_\mu \leftrightarrow \tilde{A}_\mu$
- **Linearised Gravity**
  $h_{\mu\nu} \leftrightarrow \tilde{h}_{\mu\nu} \qquad h_{\mu\nu} \leftrightarrow \tilde{h}_{\mu\nu} \leftrightarrow \tilde{\tilde{h}}_{\mu\nu}$
- Nieto 1999: Macdowell-Mansouri with Λ
- Hull 2000: Symm of lin. equns of motion
- Henneaux & Teitelboim 2005: Symm of an action written using superpotentials
- Deser & Seminara 2005: No extension to non-linear GR
- Julia,Levie,Ray 2005: Linerised in de Sitter

Mass and Dual Mass

$$R_{\mu\nu} = t_{\mu\nu}$$

$$\tilde{R}_{\mu_1 \ldots \mu_n \rho \, \nu}{}^{\rho} = \tilde{t}_{\mu_1 \ldots \mu_n \, \nu}$$

$$R_{[\mu\nu \, \sigma]\tau} = \frac{1}{n!} \epsilon_{\mu\nu \, \sigma}{}^{\mu_1 \mu_2 \ldots \mu_n} \tilde{t}_{\mu_1 \mu_2 \ldots \mu_n}$$

Mass and Dual Mass

$$R_{\mu\nu} = t_{\mu\nu} \qquad t_{\mu\nu} = T_{\mu\nu} + \frac{1}{D-2}\eta_{\mu\nu}T$$

$$\tilde{R}_{\mu_1\ldots\mu_n\rho\,\nu}{}^{\rho} = \tilde{t}_{\mu_1\ldots\mu_n\,\nu}$$

$$\tilde{t}_{\mu_1\ldots\mu_n\,\nu} = \tilde{T}_{\mu_1\ldots\mu_n\,\nu} + \frac{n}{2}\eta_{\nu[\mu_1}\tilde{T}_{\mu_2\ldots\mu_n]\rho}{}^{\rho}$$

$$R_{[\mu\nu\,\sigma]\tau} = \frac{1}{n!}\epsilon_{\mu\nu\,\sigma}{}^{\mu_1\mu_2\ldots\mu_n}\tilde{t}_{\mu_1\mu_2\ldots\mu_n}$$

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$$R_{[\mu\nu\,\sigma]\tau} = \frac{1}{n!}\epsilon_{\mu\nu\,\sigma}{}^{\mu_1\mu_2\ldots\mu_n}\tilde{t}_{\mu_1\mu_2\ldots\mu_n}$$

Just 2 kinds

Electric and Magnetic Grav Sources $T, \tilde{T}$

$\tilde{T}$: Dirac strings for h

T : Dirac strings for $\tilde{h}$

Non-Linear Gravity with Killing Vector $\frac{\partial}{\partial y}$

$g_{\mu\nu} \rightarrow (g_{mn}, g_{my}, g_{yy})$

Graviphoton in D-1 dimensions $A_m \sim g_{my}$

Dualise in D-1 dimensions: D-4 form $\tilde{A}_{m_1 \ldots m_{D-4}}$

D=4: Scalar NUT potential a.

SL(2,R) Ehlers symmetry. 2 scalars (a, g_{yy}) in $\frac{SL(2,R)}{U(1)}$

D=5: E-M duality for A,Ã

Electric charge: P^y

Magnetic charge: KK monopole

This E-M duality part of U-duality in M-theory

M-Theory Compactified on a Torus

D=4:

28 vector fields

28 electric + 28 magnetic charges

$E_7(Z)$ symmetry

D=5:

27 vector fields

27 electric charges Z^{ab} + 27 magnetic strings

$E_6(Z)$ symmetry

"Topological" charge K, carried by KK monopoles

Reduce 5→4:

Graviphoton $g_{\mu 5}$ Electric charge: P^5 Magnetic charge: K

D=5 Superalgebra

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab}\left(\Gamma^\mu C\right)_{\alpha\beta} P_\mu + C_{\alpha\beta}(Z^{ab} + \Omega^{ab} K)$$

K carried by KK monopoles Gibbons & Perry

Z^{ab} carried by charged 0-branes (from wrapped M-branes)

BPS bound

$$M \geq |K|$$

Full D=5 theory on S^1:

No killing vectors, full KK tower etc

Has $E_7(Z)$ symmetry

Includes duality $P^5 \leftrightarrow K$

D>5: D-5 form charge K carried by KK monopoles CMH

K-Charge in D=5

Spacetime M asymptotic to $\bar{M}$
k asymptotic to Killing vector on $\bar{M}$

$$\Delta\omega = \omega - \bar{\omega}$$

Difference in spin connections: Asymptotic tensor
ADM Momentum for k: Integral at spatial infinity Σ^3

$$P[k] = \frac{1}{16\pi^2}\int_{\Sigma^3} *(e^A_\wedge e^B_\wedge k)_\wedge \Delta\omega_{AB}$$

Nestor

K-charge

$$K = \frac{1}{16\pi^2}\int_{\Sigma^3} e^A_\wedge e^B{}_\wedge \Delta\omega_{AB}$$

Hull

K and NUT Charge

NUT Charge: Reduce on Killing vector
N is magnetic charge for graviphoton in D=4

KK Monopole spacetime: (Taub-NUT)x(time)
NUT charge N
S^1 fibre, asymptotically radius R=|N|

$$K=RN=N|N|$$

Gravitational Instantons Carry K

- Nx(time), N gravitational instanton

  N Gibbons-Hawking multi-instanton space with general sources.

- Metric has Dirac string singularities in general, but connection well-defined

- If all charges are equal, singularities can be removed by identifying under discrete group: ALE or ALF instanton

- String singularities be allowed in quantum gravity or M-theory? Small instantons?

Duality in Interacting Theories

D=4 N=4 SYM

Free field theory S-dual

Non-abelian field theory not S-dual

But quantum theory has SL(2,Z) symmetry:

1. AdS/CFT, SL(2,Z) from IIB
2. Physical spectrum S-dual
3. (2,0) theory on T^2 gives geometric SL(2,Z)
   6-D CFT from multiple M5-branes

D=4 N=8 SUGRA/M-theory

Free field theory has gravitational S-duality

Interacting theory?

Role for (4,0) 6-D CFT?

D=6 (2,0) free theory

R-symmetry Sp(2)=USp(4)

Superconformal OSp(4/8*) ⊃ USp(4)xSO(6,2)

$$B_{MN} \qquad\qquad H = *H$$

5 scalars, 4 fermions

Reduce to D=5

$$B_{\mu\nu}, B_{\mu 5} = A_\mu \qquad\qquad H = *F$$

A,B dual, not independent

A, 5 scalars, 4 fermions: D=5 N=4 vector multiplet

Reduce to D=4

2 vector fields $B_{\mu i} = A_{\mu i} \qquad i = 1, 2$ $F_1 = *F_2$

SL(2,Z): diffeos on T^2 (A_1, A_2) doublet

Only one independent field, D=4 N=4 vector multiplet

SL(2,Z): $(A_1, \tilde{A}_1)$ doublet, E-M duality

D=6 Free (4,0) Theory

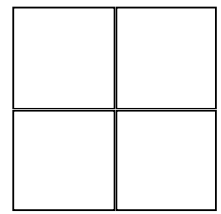
Hull

42 scalars

27 self-dual B_2: $H = *H$

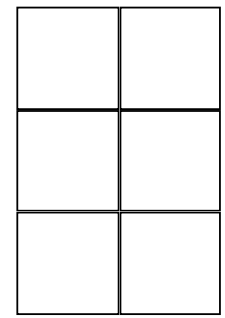
Gauge field

$$C_{MNPQ}$$



Curvature

$$G_{MNPQRS}$$



Self-dual: G=*G=G*

"Supergravity without a graviton"

Superconformal OSp(8/8*) ⊃ USp(8)xSO(6,2)

Reduce to D=5

27 $B_2 \rightarrow$ 27 vectors A_1, 42 scalars $\rightarrow$ 42 scalars

$$C_{\mu 5 \nu 5} = h_{\mu\nu}$$
$$C_{\mu\nu\rho 5} = \tilde{h}_{\mu\nu\,\rho}$$
$$C_{\mu\nu\rho\sigma} = \tilde{\tilde{h}}_{\mu\nu\rho\sigma}$$

Self-duality: Only one of these independent, dual gravitons

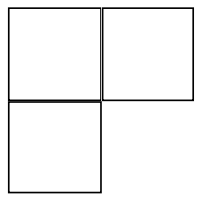
Spectrum of D=5 N=8 SUGRA!

Graviton, 27 vectors, 42 scalars Diffeos

Vectors from B_{MN}

Graviton from C_{MNPQ}

Diffeos from C gauge transformations. Parameter



Reduce to D=4

42 scalars → 42 scalars, Dual vector doublets $B_{\mu i} = A_{\mu i}$

Metrics $C_{\mu(ij)\nu} = -(h_{\mu\nu})_{ij}$

Curvatures: R_{21}=*R_{11}, R_{12} =R_{11}*, R_{22} =*R_{11}*

$$h_{21} = \tilde{h}_{11}, \qquad h_{22} = \tilde{\tilde{h}}_{11}$$

Just h_{11} independent

SL(2,Z) on torus:
(A_1,A_2) doublets, E-M duality
Triplet h_{ij}: gravitational triality

5-D SYM at Strong Coupling

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab}\left(\Gamma^\mu C\right)_{\alpha\beta} P_\mu + C_{\alpha\beta}(Z^{ab} + \Omega^{ab} K)$$

Z electric charges: carried by W-bosons etc
YM instanton in R^4 lifts to BPS soliton in 5-D
K proportional to instanton number n, (2,0) short mult.

$$M \propto \frac{n}{g^2_{YM}}$$

Light at strong coupling: KK tower for 6'th dimension
Decompactifies to (2,0) theory in 6D as $g^2_{YM} \to \infty$

Witten, Rozali

(2,0) Interacting CFT

D=5 non-renormalizable, defined within string theory
e.g. D4 brane theory
Strong coupling limit defined within string theory
e.g. multiple D4 branes → multiple M5 branes
No direct construction of interacting (2,0) theory.
Reduce on T^2 gives interacting N=4 SYM
and SL(2,Z) S-duality from torus diffeos

g_{YM} dimensionful. Limit is one to high energies

$$E >> (g_{YM})^{-2} \qquad\qquad E(g_{YM})^2 \to \infty$$

5-D SUGRA at Strong Coupling

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab}\left(\Gamma^\mu C\right)_{\alpha\beta} P_\mu + C_{\alpha\beta}(Z^{ab} + \Omega^{ab} K)$$

K carried by gravitational instantons in 4D, lifted to KK solitons in 5D. (4,0) short multiplet

$$M \propto \frac{n}{l_{Plank}}$$

Become Light in strong coupling limit $l_{Plank} \to \infty$
Decompactification limit with instantons as a KK tower?
If so, must decompactify to a (4,0) theory in 6D

Decompactification?

- If theory has states carrying charge K, they fit into right supermultiplet for KK modes of (4,0) theory
- K=…-2,-1,0,1,2,…, then they can be associated with KK modes
- These become light at high energies — decompactification to (4,0) theory in D=6
- Theory must decompactify (if limit exists)

(2,0) & (4,0) 6-D CFTs

- No local covariant interacting field theory
- D=5 BPS electric 0-branes and magnetic strings lift to self-dual strings in D=6. Tension to zero in conformal limit
- Large superconformal symmetry: (4,0) has 32+32 susys
- YM and graviton in D=5 lift to self-dual tensor gauge fields
- D=5 g_{YM} & l_{planck} from R_6 as no scale in 6-D

Symmetry of (4,0)

Free theory:

Conventional field theory in flat background

Background diffeomorphisms + gauge trans

$$\delta C_{MN\,PQ} = \partial_{[M}\chi_{N]PQ} + \partial_{[P}\chi_{Q]MN} - 2\partial_{[M}\chi_{NPQ]}$$

Reduce to D=5 or D=4:

Combine $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

2 Symmetries are the same for $g_{\mu\nu}$

On T^2, background diffeos give SL(2,Z) S-duality of both spin-1 and spin-2 fields in D=4

Interacting D=6 theory:

Can't combine background η_{MN} & field C_{MNPQ}

Don't expect D=6 diffeos, but exotic symmetries that give D=5 diffeomorphisms

Without D=6 diffeomorphisms, no reason to expect SL(2,Z) and hence no "derivation" of gravitational S-duality (unlike free case)

Without D=6 diffeomorphisms, should spacetime be replaced by something more exotic?
This should be consistent with free limit being a conventional field theory

Conclusions

- Dual gravitons and gravitational S-duality work well for free theory, or in full theory with Killing vectors
- For $D \geq 5$, charge K carried by KK monopoles, and branes from D=4 instantons. Related to NUT charge and magnetic charge of KK monopoles
- For D=4 SYM or linearised SUGRA, S-duality from (2,0) or (4,0) theory on T^2

- Possibility of (4,0) theory as a limit of M-theory. Corner of M-theory with vast symmetry and unusual features

- Not a conventional spacetime theory, metric & diffeos replaced

- Is (4,0) CFT a decoupling limit of (4,0) sector of M-theory?

- Implications for gravity and M-theory?