ON SUPERSYMMETRIC REGULARIZATION in FIELD THEORY and HOLOGRAPHY

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Supersymmetric theories, dualities and deformations

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Based on: 1606.02724 with Benetti Genolini, Martelli, Sparks + to appear

+ previous work with Assel, Di Pietro, Komargodski, Lorenzen, Martelli

Supersymmetric regularization

- Recently: many new exact results in supersymmetric QFT mainly thanks to **localization**
- need to renormalize UV divergences

  ➜ crucial to use a regularization scheme that preserves supersymmetry

  - One way to assess this : check susy Ward identities at the end

- Even if regularization is supersymmetric, there may be ambiguities,

  ➜ important to classify them in order to extract physically meaningful result

Example — Gerchkovitz, Gomis, Komargodski

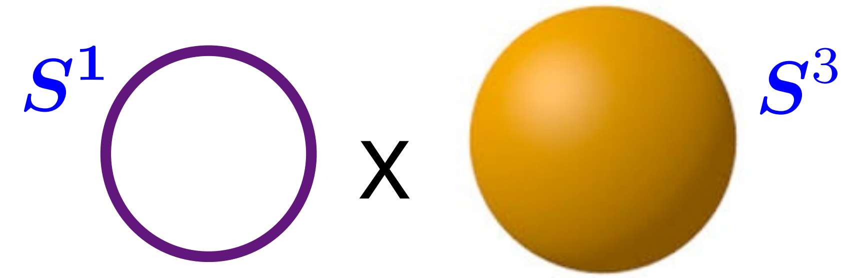
Partition function of N=2 SCFT's on S^4 as a function of marginal couplings $\tau, \bar\tau$ computes the Kähler potential $K(\tau,\bar\tau)$ on the conformal manifold.
Ambiguous by $F(\tau) + \overline{F}(\bar\tau)$: correspond to Kähler transformations.

Field theory side

- Consider a 4d, N=1 SCFT.

  can be placed on a space with topology

  preserving some susy



  $\longleftrightarrow$ Hopf surfaces $\mathcal{H}_{p,q}$ with complex structure parameters p, q

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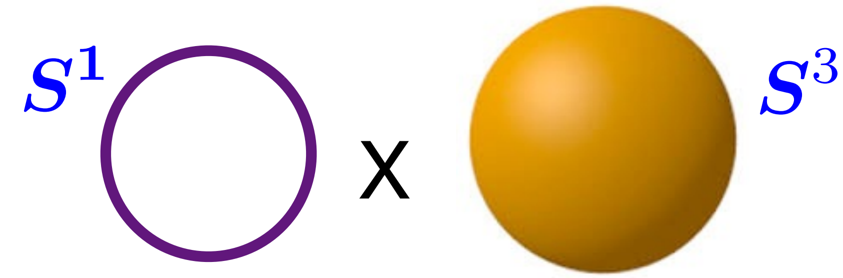

  ⟷ Hopf surfaces $\mathcal{H}_{p,q}$ with complex structure parameters p, q

- Supersymmetric partition function $Z[\mathcal{H}_{p,q}]$ does not depend on Hermitian metric and is a holomorphic function of the complex structure parameters

Closset, Dumitrescu, Festuccia, Komargodski

susy Ward identities : $\delta_{\text{Hermitian}}(-\log Z) = 0$

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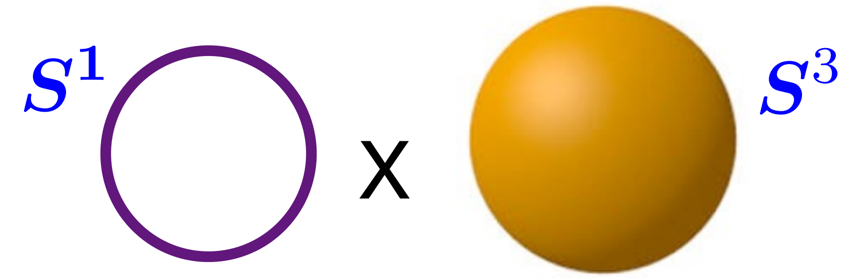

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- Localization yields: $Z[\mathcal{H}_{p,q}] = \mathrm{e}^{-\mathcal{F}(p,q)}\mathcal{I}(p,q)$ Assel, D.C., Martelli

  only depends on p, q
  although we allowed for a general metric

  ✔ susy Ward identities

  $\mathcal{I}(p,q)$ → superconformal index

  $$\mathcal{I}(p,q) = \mathrm{tr}\left[(-1)^F p^{J+J'-\frac{R}{2}} q^{J-J'-\frac{R}{2}}\right]$$

  p, q ← fugacities

supersymmetric Casimir energy

Assel, D.C., Martelli

$$Z[\mathcal{H}_{p,q}] = \mathrm{e}^{-\mathcal{F}(p,q)}\mathcal{I}(p,q)$$

superconformal index

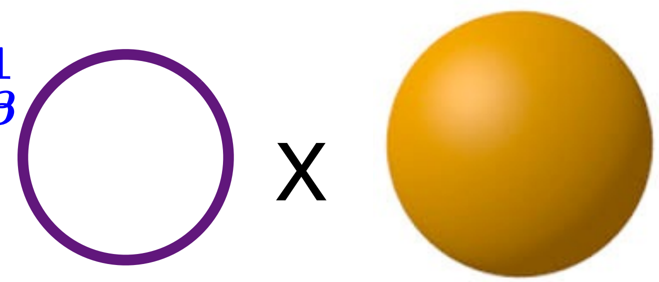
$$\mathcal{F}(p,q) = \beta E_{\text{susy}}(b_1, b_2) \qquad p = \mathrm{e}^{-\beta b_1}, \quad q = \mathrm{e}^{-\beta b_2}$$

$$E_{\text{susy}} = \frac{2}{3}(b_1 + b_2)(a - c) + \frac{2}{27}\frac{(b_1+b_2)^3}{b_1 b_2}(3c - 2a)$$

supersymmetric Casimir energy

a, c central charges

$$Z = \int \mathcal{D}\phi \, e^{-S[\phi]} = \mathrm{Tr}\, e^{-\beta H}$$

S^1_β  S^3

$Z \sim e^{-\beta E_{\text{Casimir}}}$ as $\beta \to \infty$ (projects to ground state)

The importance of being supersymmetric

- $Z_{\text{non-susy}}$ is ambiguous due to local counterterm :

$$-\log Z_{\text{non-susy}} + \underbrace{b \int d^4x \sqrt{g}\, R^2}_{\neq 0 \text{ on } S^1 \times S^3}$$

➜ Casimir energy on $S^1 \times S^3$ is scheme-dependent

- **In susy case :**

ambiguities are gauge-invariant (new minimal) supergravity actions of dim = 4

◆ they are all F-terms

◆ F-terms vanish on susy backgrounds with 2 supercharges of opposite R-charge

$$-\log Z_{\text{susy}} + \underbrace{b \int d^4x \sqrt{g}\, \left[(R + 6 V_\mu V^\mu)^2 - 8 F_{\mu\nu} F^{\mu\nu} \right] + \text{fermions}}_{= 0 \text{ on } S^1 \times S^3 \,!}$$

The importance of being supersymmetric

- If an N=1 SCFT is defined on a supersymmetric background preserving two Euclidean supercharges, and using new minimal sugra

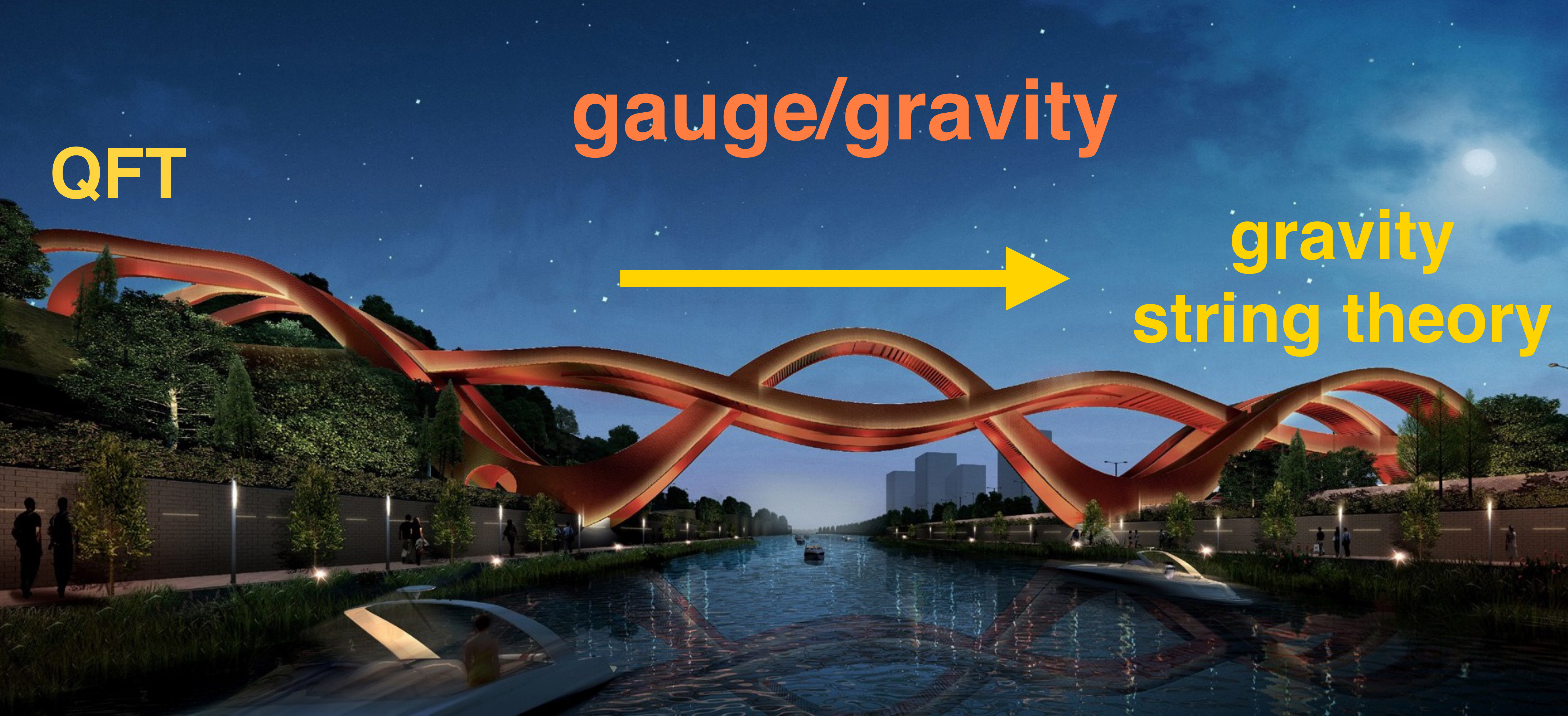
- then the dependence of the supersymmetric partition function on the background is free of ambiguities

$$\rightarrow \quad E_{\text{susy}} = \frac{2}{3}(b_1 + b_2)(a - c) + \frac{2}{27}\frac{(b_1 + b_2)^3}{b_1 b_2}(3c - 2a)$$

is an intrinsic observable of the SCFT

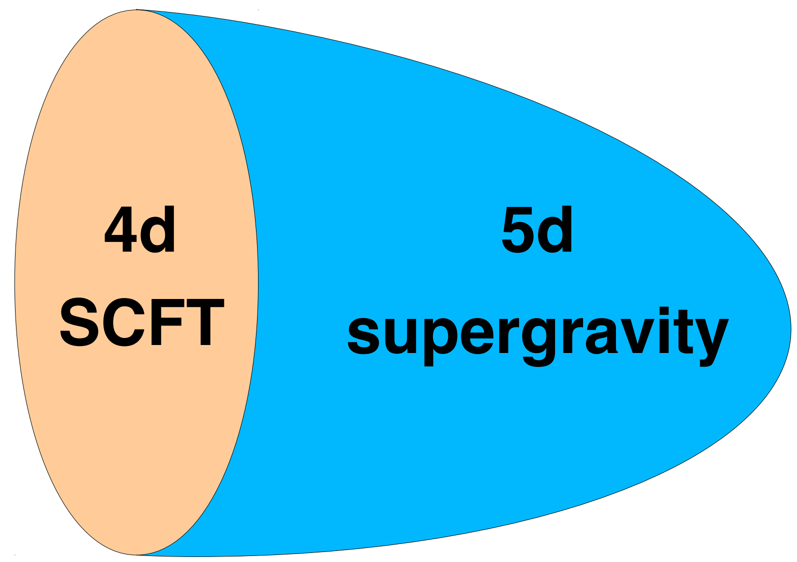
- superalgebra implies the BPS condition $E_{\text{susy}} = \frac{1}{r}\langle R \rangle$ on round $S^1_\beta \times S^3_r$

→ **vacuum charged!**



Credit: Next Architects

Holographic dictionary



	M_4	$=$	∂M_5
sources for e.m. tensor multiplet			boundary conditions for gravity multiplet
	$g,\ A - \frac{3}{2}V$		$g^{\mathrm{bulk}},\ A^{\mathrm{bulk}}$ (minimal is enough)
at large Nc	$-\log Z[M_4]$	$=$	$S_{\text{on-shell}}[M_5]$
central charges	$c = a$	$=$	$\pi^2 \ell^3/\kappa_5^2$ 5d grav. coupling
Casimir energy			grav. energy of a solution dual to CFT vacuum
R-charge			charge of the solution under graviphoton

match??

Holographic Casimir energy

$$Z[\mathcal{H}_{p,q}] = e^{-\mathcal{F}(p,q)}\mathcal{I}(p,q)$$

superconformal index

$$\mathcal{F}(p,q) = \beta E_{\rm susy}(b_1, b_2)$$

$$E_{\rm susy} = \frac{2}{3}(b_1+b_2)(a-c) + \frac{2}{27}\frac{(b_1+b_2)^3}{b_1 b_2}(3c-2a)$$

dominates Z at large Nc ➜ **prediction for dual supergravity solutions**

- There should be a family of susy solutions with $\partial M_5 = \mathcal{H}_{p,q}$ such that :

$$S_{\rm 5d\, sugra}[M_5] = \frac{2}{27}\beta\frac{(b_1+b_2)^3}{b_1 b_2}\frac{\pi^2\ell^3}{\kappa_5^2}$$

$$c = a = \pi^2\ell^3/\kappa_5^2$$

Holographic Casimir energy

$$Z[\mathcal{H}_{p,q}] \;=\; \mathrm{e}^{-\mathcal{F}(p,q)}\mathcal{I}(p\,,q)$$

superconformal index

$$\mathcal{F}(p,q) = \beta E_{\text{susy}}(b_1\,, b_2)$$

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$$S_{\text{5d sugra}}[M_5] \;=\; \frac{2}{27}\beta\frac{(b_1+b_2)^3}{b_1 b_2}\frac{\pi^2\ell^3}{\kappa_5^2} \;=\; \frac{16}{27}\frac{\beta}{r}\frac{\pi^2\ell^3}{\kappa_5^2}$$

$$c = a = \pi^2\ell^3/\kappa_5^2$$

$$b_1 = b_2 = \frac{1}{r}$$ round $S^1_\beta \times S^3_r$

Holographic renormalization

On-shell action a priori divergent

$$S = \frac{1}{2\kappa_5^2}\int_{M_5} d^5x\sqrt{g}\left(R[g] - F^2 + \frac{12}{\ell^2}\right) - A\wedge F\wedge F$$

5d supergravity action $S_{\mathbf{bulk}}$

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5d supergravity action $S_{\mathbf{bulk}}$

$$+ \frac{1}{\kappa_5^2}\int_{\partial M_5} d^4x\sqrt{h}\left(K - \frac{3}{\ell} - \frac{\ell}{4}R[h]\right)$$

divergent counterterms

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divergent counterterms

$$+ \frac{1}{\kappa_5^2}\int_{\partial M_5} d^4x\sqrt{h}\left(\lambda_1 R_{ijkl}[h]^2 + \lambda_2 R_{ij}[h]^2 + \lambda_3 R[h]^2 + \lambda_4 F_{ij}^2\right)$$

finite boundary terms ➜ parameterize different schemes

Holographic renormalization

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finite boundary terms ➜ parameterize different schemes

- For round $S^1_\beta \times S^3_r$ the most natural candidate is pure Anti de Sitter space

Choosing $\lambda_1 = \lambda_2 = \lambda_3 = 0$

$$S = \frac{3}{4}\frac{\beta}{r}\frac{\pi^2\ell^3}{\kappa_5^2} \neq \frac{16}{27}\frac{\beta}{r}\frac{\pi^2\ell^3}{\kappa_5^2}$$

does **NOT match** the susy QFT result!

…could think about adjusting the λ's … but …

Holographic renormalization

- $$\frac{1}{\sqrt{g^{\rm bdy}}}\frac{\delta S}{\delta A_i^{\rm bdy}} = j^i$$ holographic R-current ➜ holographic R-charge $\langle R \rangle$

  however unavoidably for $\mathbf{AdS_5}$: $\langle R \rangle = 0$

➜ $\langle E \rangle \neq \frac{1}{r}\langle R \rangle$ BPS relation violated

unless one tunes λ's such that

$\langle E \rangle = \frac{1}{r}\langle R \rangle = 0$ misses Casimir energy

…something is not working here

New supergravity solutions

Benetti Genolini, D.C., Martelli, Sparks

We constructed a very general AlAdS solution perturbatively near the boundary

$$g^{\rm bulk} = \frac{d\rho^2}{\rho^2} + \frac{1}{\rho^2}\left[g^{\rm bdy} + g^{(2)}\rho^2 + \left(g^{(4)} + \tilde{g}^{(4)}\log\rho^2\right)\rho^4 + \ldots\right]$$

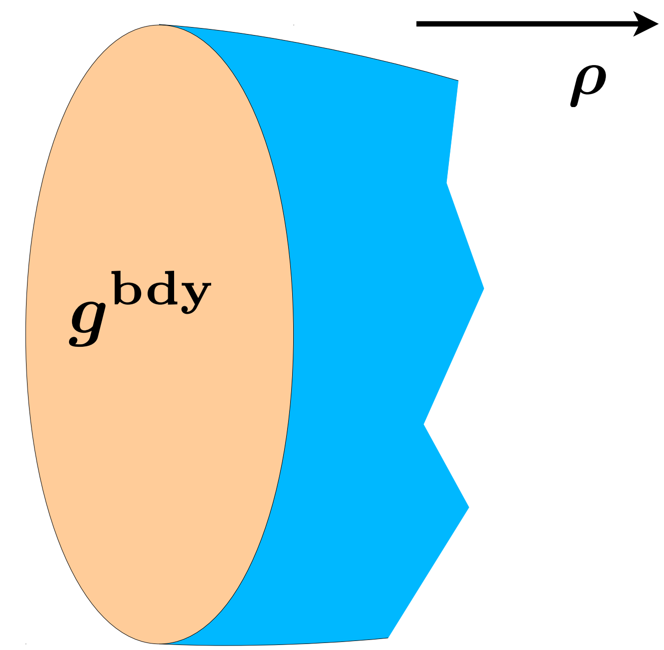
- boundary metric:

$$g^{\rm bdy} = d\tau^2 + (d\psi + a)^2 + 4e^w dz d\bar{z}$$

with $da = i\, u\, e^w dz \wedge d\bar{z}$

$u(z,\bar{z})$, $w(z,\bar{z})$ arbitrary

describes $S^1 \times S^3$ topology (and more)



- graviphoton field $A^{\rm bulk}$ also determined

$$A^{\rm bdy} = -\frac{1}{\sqrt{3}}\left[\frac{i}{8}u\, d\tau + \frac{1}{4}u(d\psi + a) + \frac{i}{4}(\partial_{\bar{z}}w\, d\bar{z} - \partial_z w\, dz) + \gamma d\psi\right]$$

- 4 free subleading functions $k_1(z,\bar{z})$, $k_2(z,\bar{z})$, $k_3(z,\bar{z})$, $k_4(z,\bar{z})$

New supergravity solutions

- non-trivial:

  susy involves solving 6th-order equation for auxiliary Kähler metric

$$\nabla^2\left(\frac{1}{2}\nabla^2 R + \frac{2}{3}R_{pq}R^{pq} - \frac{1}{3}R^2\right) + \nabla^m(R_{mn}\partial^n R) \;=\; 0$$

D.C., Lorenzen, Martelli

⚠ bulk metric has complex components (but real in Lorentzian signature)

⚠ on-shell action gauge-dependent due to Chern-Simons term $\int A\wedge F\wedge F$

→ crucial to fix the gauge properly

- Killing spinors independent of time
- gauge field globally well-defined

Supersymmetric holographic renormalization

Vary the boundary data keeping complex structure on ∂M fixed

$\longleftrightarrow$ vary the functions $u(z,\bar{z})$, $w(z,\bar{z})$ with globally def. variations

$$\delta S = \int_{M_4} \mathrm{d}^4 x \sqrt{g^{\mathrm{bdy}}} \left(\frac{1}{2} T^{ij} \delta g_{ij}^{\mathrm{bdy}} + j^i \delta A_i^{\mathrm{bdy}} \right)$$

⚠ There is no choice of $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ such that $\delta_u S = 0 = \delta_w S$

➜ **Holographic renormalization violates field theory susy Ward identities!**

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new boundary term ΔS_{new} such that $S_{\mathrm{susy}} = S + \Delta S_{\mathrm{new}}$ satisfies $\delta S_{\mathrm{susy}} = 0$

$$\Delta S_{\mathrm{new}}[M_4] = \frac{1}{\kappa_5^2} \int_{M_4} (\mathrm{i} A^{\mathrm{bdy}} \wedge \Phi + \Psi)$$

S is taken with $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$

$$\Phi = \frac{1}{72\sqrt{3}} \left(u^3 + 4u \Box w \right) \mathrm{i}\, \mathrm{e}^w \mathrm{d}z \wedge \mathrm{d}\bar{z} \wedge (2\, \mathrm{d}\psi + \mathrm{i}\, \mathrm{d}\tau)$$

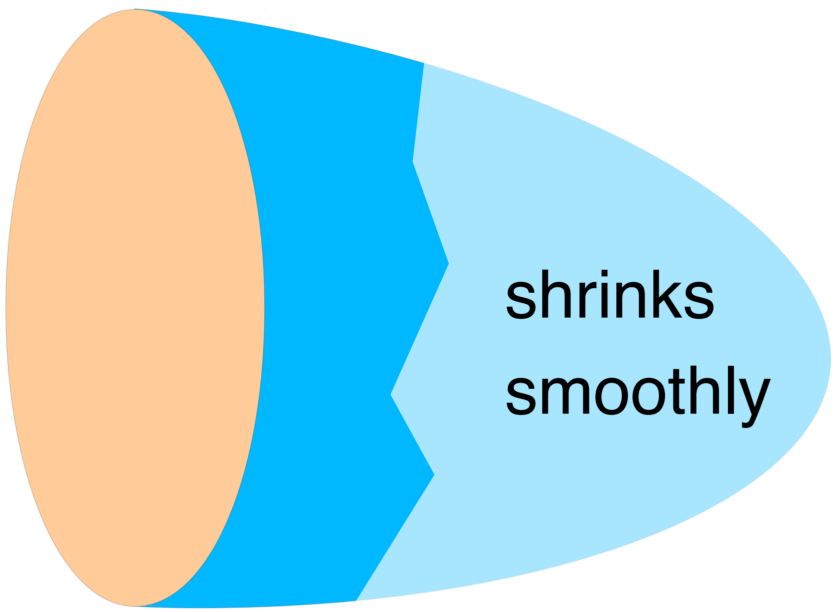
$$\Psi = \frac{1}{2^{11} 3^2} \left(-19 u^4 - 48 u^2 \Box w \right) \mathrm{d}^4 x \sqrt{g^{\mathrm{bdy}}}$$

Supersymmetric holographic renormalization

We propose that $S_{\rm susy}[M_5] = S[M_5] + \Delta S_{\rm new}[M_4]$

should be identified with the SCFT $-\log Z_{\rm susy}[M_4]$

Although we don't know the solution in the interior, under topological assumption
we can actually evaluate the on-shell action



- $S_{\rm bulk}$ reduces to a boundary term
- 4 subleading functions $k_1(z,\bar{z}),\ k_2(z,\bar{z}),\ k_3(z,\bar{z}),\ k_4(z,\bar{z})$ drop out !

Supersymmetric holographic renormalization

$$S_{\rm susy} \;=\; S + \Delta S_{\rm new} \;=\; \frac{\gamma^2}{27}\int_{M_4} {\rm d}^4 x\,\sqrt{g^{\rm bdy}}\,R_{(2d)}\,\frac{\ell^3}{\kappa_5^2}$$

Supersymmetric holographic renormalization

$$8\pi^2 \beta \frac{b_1 + b_2}{b_1 b_2}$$

$$S_{\text{susy}} = S + \Delta S_{\text{new}} = \frac{\gamma^2}{27} \underbrace{\int_{M_4} \mathrm{d}^4 x \sqrt{g^{\text{bdy}}} R_{(2d)}}_{} \frac{\ell^3}{\kappa_5^2}$$

Supersymmetric holographic renormalization

from $A^{\mathrm{bdy}} = \ldots + \gamma\, \mathrm{d}\psi$

$$\gamma = \tfrac{1}{2}(b_1 + b_2) \qquad\qquad 8\pi^2 \beta \, \frac{b_1 + b_2}{b_1 b_2}$$

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$$= \frac{2}{27} \beta \frac{(b_1 + b_2)^3}{b_1 b_2} \frac{\pi^2 \ell^3}{\kappa_5^2}$$

✔ $\beta\, E_{\mathrm{susy}}$

also computed $\langle R \rangle$ via $\dfrac{1}{\sqrt{g^{\mathrm{bdy}}}} \dfrac{\delta S_{\mathrm{susy}}}{\delta A_i^{\mathrm{bdy}}} = j^i$ ✔ BPS relation

Conclusions

- Standard holographic renormalization in 5d violates susy
- Identified boundary terms $\Delta S_{\rm new}$ that restore susy Ward identities
- Constructed asymptotic solutions such that

$$S_{\rm susy} = S + \Delta S_{\rm new} = \frac{2}{27}\beta \frac{(b_1+b_2)^3}{b_1 b_2}\frac{\pi^2 \ell^3}{\kappa_5^2}$$

✔ field theory

? $\Delta S_{\rm new}$ in covariant form?

? which boundary fields involved?

First principle derivation such that bulk + boundary action is supersymmetric?

- does $\Delta S_{\rm new}$ have consequences for susy AdS5 black holes?

➜ Zaffaroni's talk

... thank you !