On the Chiral Ring of Warped AdS(3) Compactifications of Type IIB Supergravity

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Supersymmetric Theories, their Dualities and Deformations
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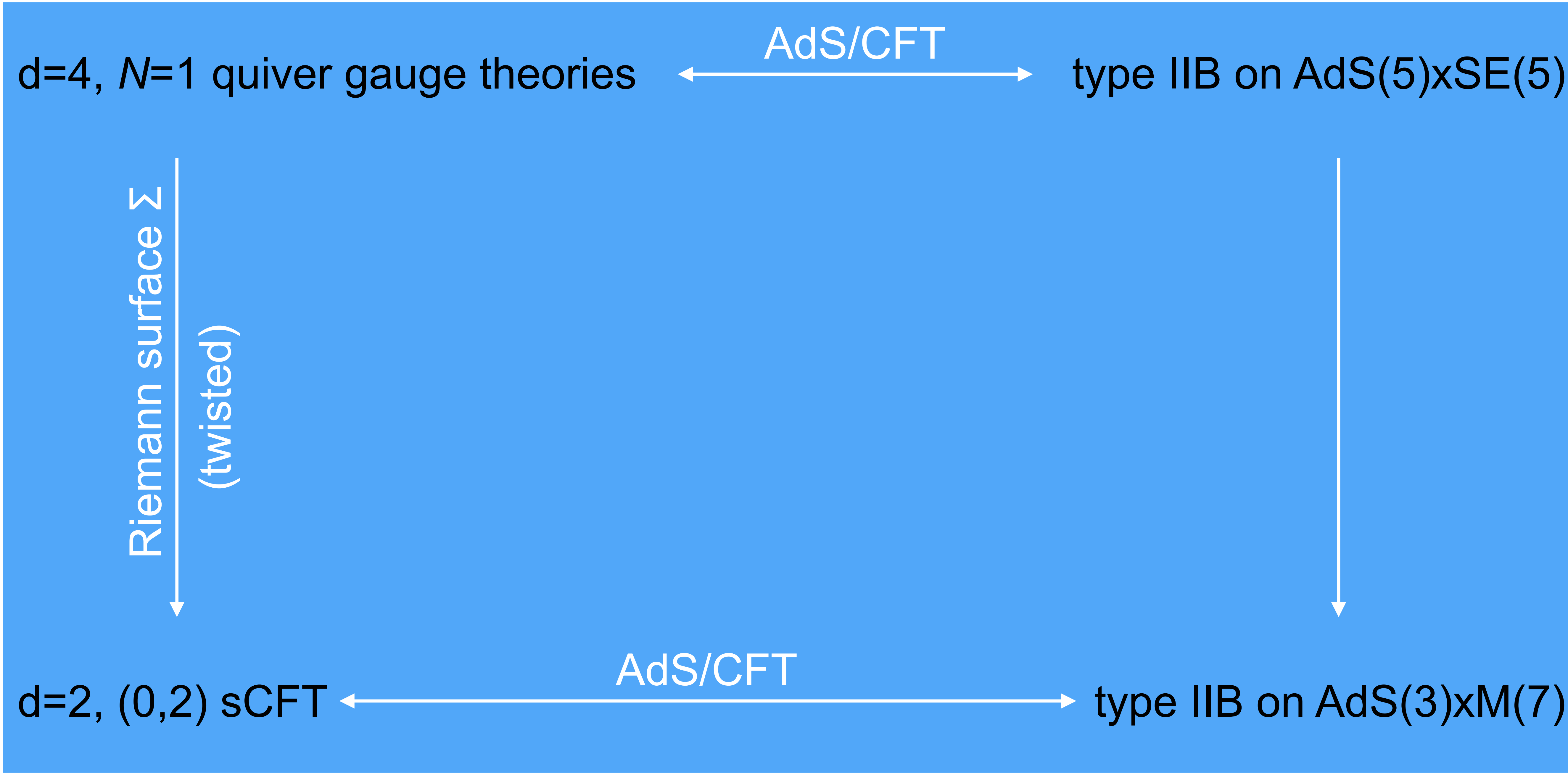
Based on work with Orestis Vasilakis.

Motivation I: 4d Theories on Riemann Surfaces



Obvious questions: central charge, **spectrum/chiral ring**, 3-pt. functions.

Benini, Bobev, Crichigno

Motivation II: Type IIB on AdS(3) w/ F_5

Generalisation: Warped AdS_3 with F_5 and dilaton

$$ds^2_{IIB} = e^{2A}\left[ds^2(\mathrm{AdS}_3) + ds^2_7\right], \qquad ds^2_7 = e^{-4A} ds^2_6 + \eta^2 .$$

$$F_5 = (1+\star)\mathrm{vol}(\mathrm{AdS}_3)\wedge F, \qquad F = \frac{1}{2}J - \frac{1}{4}d\left(e^{4A}\eta\right).$$

Generalisation of Sasaki-Einstein case:

- M_6 is Kähler.
- M_7 is Cauchy-Riemann (CR).
- $M_8=C(M_7)$ is a complex variety (not Kähler!).

Kim; Gauntlett; Donos, Mac Conamhna, Mateos, Sparks, Waldram, …; Harvey, Lawson; Yau

Motivation III: Learn about AdS/CFT from KK ↔ learn about KK from AdS/CFT.

AdS/CFT and Kaluza-Klein

Recall

scaling dimensions ↔ AdS(3) masses ↔ spectra of differential operators on M_7

Need to analyse on M_7 (among others):

$$\mathcal{L}_0 = \Delta - 8 g^{\kappa\lambda} \partial_\kappa A \partial_\lambda ,$$

spin-2 & axio-dilaton fluctuations

$$\mathcal{L}_{1/2} = \Gamma^\mu \nabla_\mu + \frac{9}{2} \Gamma^\mu \partial_\mu A + \frac{1}{2} e^{-4A} \Gamma^{\mu\nu} F_{\mu\nu} .$$

dilatino fluctuations

Compare: Klebanov, Pufu, Rocha; Bachas, Estes; Ahn, Woo; Passias, Tomasiello; …

Aim: Solve these by exploiting superconformal symmetry.

- Unitarity bounds — bounds on spectra.
- Ring structure — cohomology groups at bound.
- Supersymmetry — maps between spectra of different operators.

For Sasaki-Einstein case, see: Eager, Schmude, Tachikawa

A Subtlety: The Baryonic Contribution to the R-symmetry

From supergravity classification of background & Killing spinors

$$\eta \leftrightarrow U(1)_R$$

c-extremization yields baryonic contribution to d=2 R-symmetry

$$T_{\text{tr}} = \epsilon_1 T_1 + \epsilon_2 T_2 + T_R + \underline{\epsilon_B T_B}$$

Benini, Bobev, Crichigno

Supergravity fluctuations are intrinsically “mesonic” and not sensitive to this.

The Deformed Laplacian I — Deformed Adjoint Operators

Recall the de Rham-Laplacian:

$$(\alpha, \beta) \equiv \int \star\bar{\alpha} \wedge \beta, \qquad (d\alpha, \beta) = (\alpha, d^*\beta), \qquad \Delta = dd^* + d^*d.$$

$$(\alpha, \beta)_c \equiv \int e^{cA} \star \bar{\alpha} \wedge \beta, \qquad (d\alpha, \beta)_c = (\alpha, d_c^*\beta), \qquad \mathcal{L}_0 = dd_8^* + d_8^*d.$$

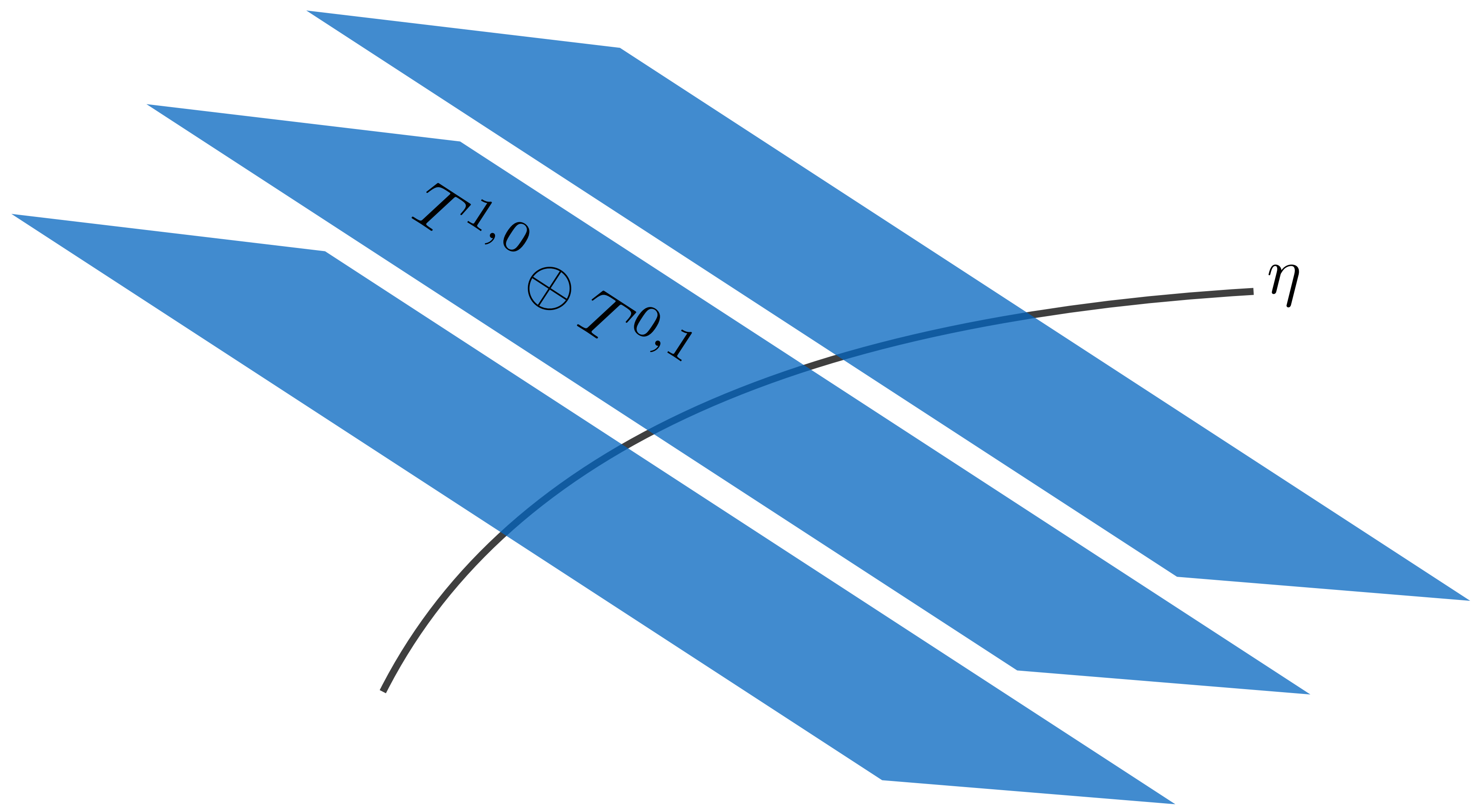
Generalises to p-forms (gauge-fixing, Hodge decomposition).

Recall that for Kähler manifolds:

$$\Delta = 2\Delta_{\bar{\partial}}, \qquad \Delta Y = 0 \text{ iff } \bar{\partial}Y = 0.$$

The Deformed Laplacian II — The CR Structure

$$T_{\mathbb{C}}M_7 = T^{1,0} \oplus T^{0,1} \oplus \mathbb{C}\eta.$$



$$[T^{1,0}, T^{1,0}] \subseteq T^{1,0} \quad \Rightarrow \quad d = \partial_b + \bar{\partial}_b + \eta \wedge \pounds_\eta.$$

Holomorphy in the CR sense is related to holomorphy on the variety M_8.

The Deformed Laplacian III — Solutions at the Bound

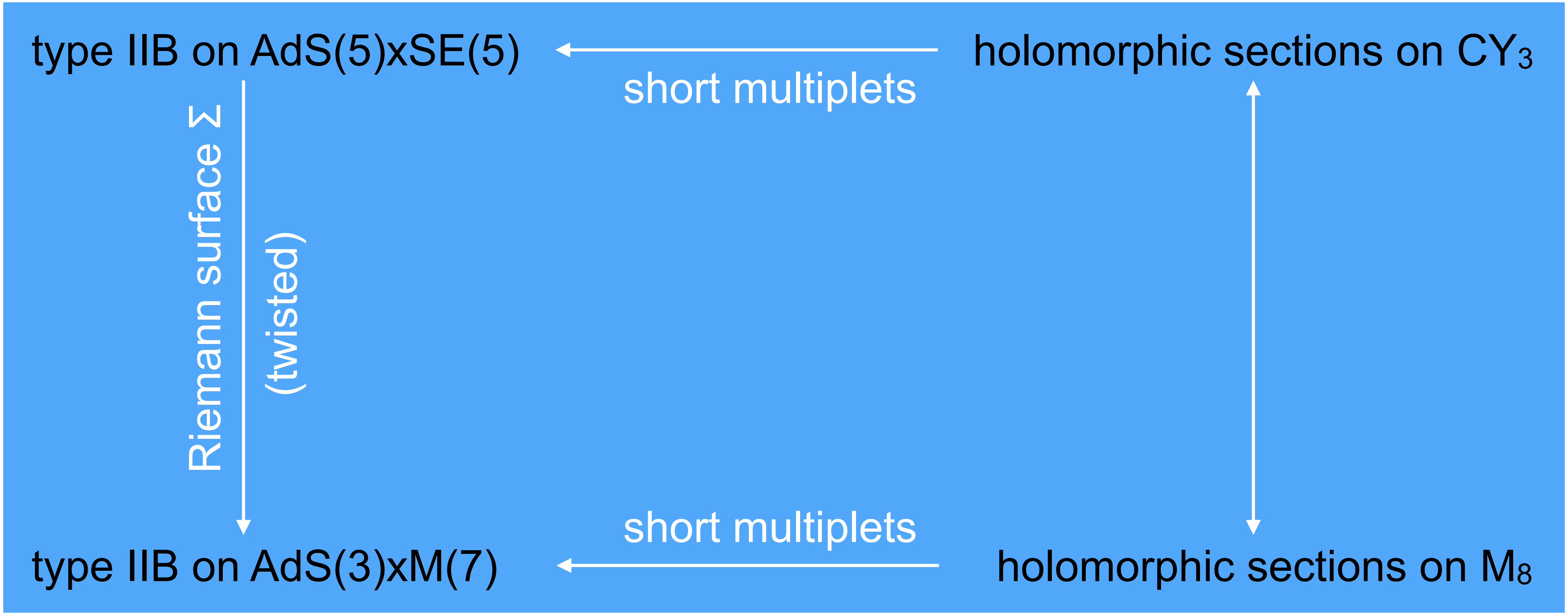
The unitarity bound for scalar "wavefunctions" (both spin-2 & axio-dilaton)

$$\pounds_\eta Y = \imath q Y, \qquad \mathcal{L}_0 Y \equiv (E_0^2 + 2E_0)Y \geq (q^2 + 2q)Y, \qquad \text{with "=" iff } \bar{\partial}_b Y = 0.$$

Thus: **Every holomorphic function** defines $Y \in H^{0,0}_{\bar{\partial}_b}(M_7) \cong H^{0,0}(M_8)$

graviton, spin 2	$\left\| \frac{E_0+4}{2} \right\rangle \otimes \left\| \frac{E_0}{2}, q \right\rangle$	at bound: chiral primary
axio-dilaton, spin 0	$\left\| \frac{E_0+2}{2} \right\rangle \otimes \left\| \frac{E_0+2}{2}, q \right\rangle$	at bound: descendant of chiral primary

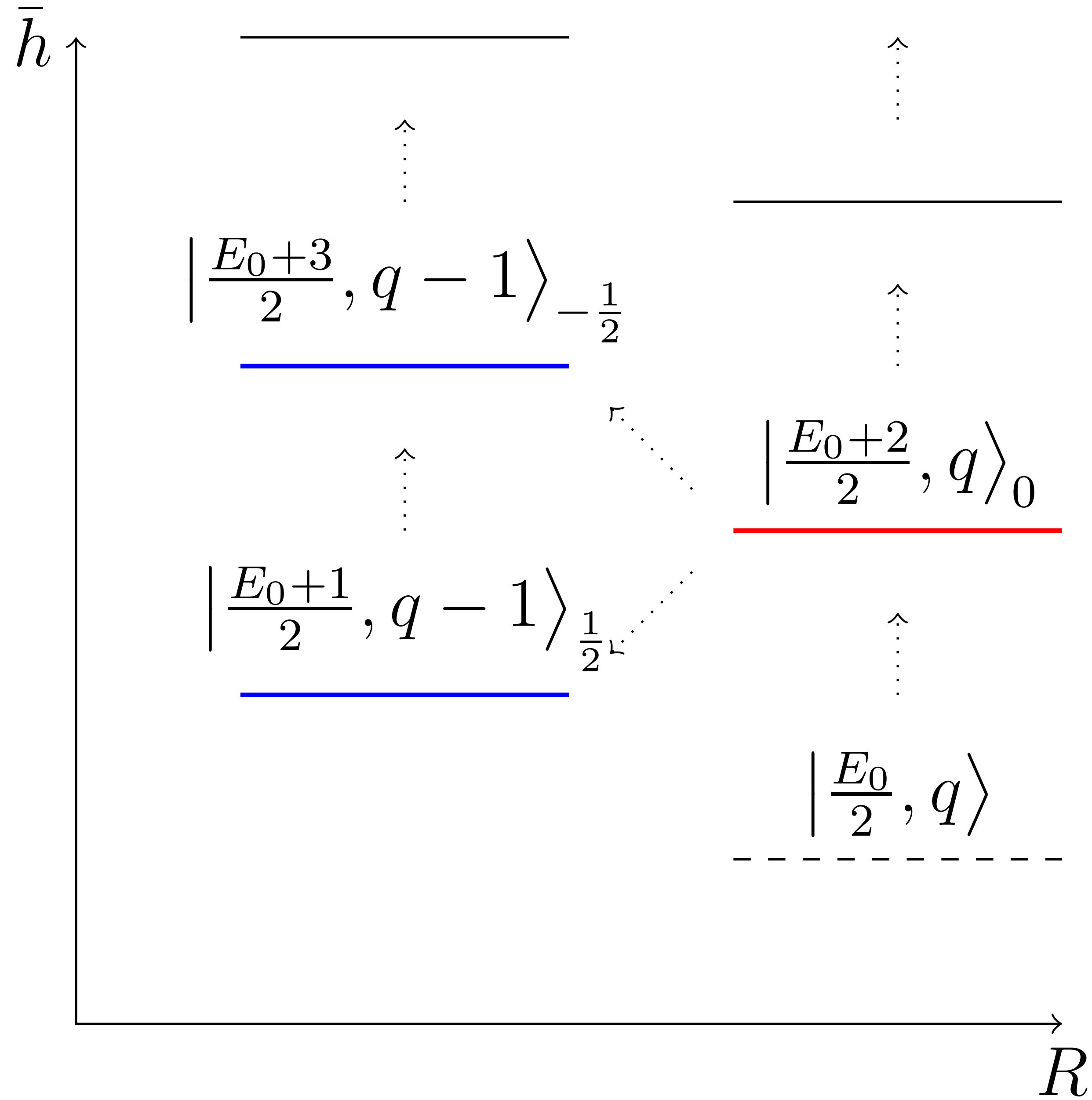
Holomorphic Sections on CY_3 and M_8.



How is M_8 related to CY_3 and Σ?

Dilatino Fluctuations & Supersymmetry

$$\mathcal{L}_{1/2}\begin{pmatrix} e^{-A}Y\zeta^c \\ e^{-A}\Gamma^\mu\partial_\mu Y\zeta^c\end{pmatrix} = \begin{pmatrix} \frac{\imath}{2} & 1 \\ -\mathcal{L}_0 & -\frac{3\imath}{2}\end{pmatrix}\begin{pmatrix} e^{-A}Y\zeta^c \\ e^{-A}\Gamma^\mu\partial_\mu Y\zeta^c\end{pmatrix}$$



Recall:

$$\mathcal{L}_0 = \Delta - 8g^{\kappa\lambda}\partial_\kappa A\partial_\lambda, \qquad \mathcal{L}_{1/2} = \Gamma^\mu\nabla_\mu + \frac{9}{2}\gamma^\mu\partial_\mu A + \frac{1}{2}e^{-4A}\gamma^{\mu\nu}F_{\mu\nu}.$$

Example I: AdS(3) x S^3

$$\mathrm{AdS}_3 \times S^3 \times T^4 \quad \rightarrow \quad C(S^3 \times T^4) = \mathbb{C}^2 \times T^4 \quad \rightarrow \quad H^{0,0}(\mathbb{C}^2)$$

Chiral primaries ↔ homogeneous polynomials; R-charges ↔ degrees

Note: T^4 is irrelevant due to holomorphy, not scaling.

Compare: Maldacena, Strominger; de Boer

Example II: The "Universal Flow" from AdS(5)x$Y^{p,q}$

"Universal flow": $U(1)_{UV} = U(1)_{IR}$

$$ds^2_{10} = ds^2(\mathrm{AdS}_3) + \frac{3}{4}ds^2(\Sigma_{g>1}) + \frac{9}{4}ds^2(\tilde{Y}^{p,q}),$$

$$ds^2(\Sigma_{g>1}) = ds^2(\mathbb{H}_2/\Gamma) = \frac{1}{x_2^2}(dx_1^2 + dx_2^2),$$

$$ds^2(\tilde{Y}^{p,q}) = \frac{1-cy}{6}(d\theta^2 + \sin^2\theta d\phi^2) + \frac{1}{wq}dy^2 + \frac{wq}{36}(d\beta + \cos\theta d\phi)^2$$
$$+ \frac{1}{9}\left[d\psi - \cos\theta d\phi + y(d\beta + c\cos\theta d\phi) - \frac{dx_1}{x_2}\right].$$

Assume separability:

$$Y = e^{\imath N_{\tilde{\psi}}\tilde{\psi} + \imath N_\phi \phi + \imath \frac{N_\alpha}{l}\alpha} X(x_1, x_2)\Theta(\theta)R(y).$$

Impose holomorphy:

$$\bar{\partial}_b Y = 0 \Rightarrow \begin{cases} \Theta(\theta) = \frac{(\sin\theta)^{N_\phi + N_{\tilde{\psi}}}}{(1+\cos\theta)^{N_\phi}} \\ R(y) = \prod_{i=1}^{3}(y - y_i)^{a_i} \end{cases}$$

See also: Ardehali, Liu, Szepietowski

Example II: The “Universal Flow” and the Role of $\Sigma_{g>1}$

For the upper half plane

$$X(z,\bar{z}) = f(z)(\mathrm{Im}z)^{-q/2}$$

For compact Σ: e.g. *Klein Quartic*

$$\Gamma = \{M \in \mathrm{PSL}(2,\mathbb{Z}) | M \equiv 1 \mod 7\}$$

$$T_7 : z \mapsto z+7, \qquad U_7 : z \mapsto \frac{z}{7z+1}$$

$$X(z,\bar{z}) = (\mathrm{Im}z)^{-q/2} \sum_{k\in\mathbb{Z}} a_k e^{\frac{2\pi\imath z}{7}}$$

$$X \circ U_7(z,\bar{z}) = (\mathrm{Im}z)^{-q/2} \left\{ [(7\mathrm{Re}z+1)^2 + (\mathrm{Im}z)^2]^{q/2} \sum_{k\in\mathbb{Z}} a_k e^{\frac{2\pi\imath z}{7(7z+1)}} \right\}$$

Only one mode survives the projection → dual to energy-momentum tensor

$$\Rightarrow q = 0, X = \mathrm{const.} \qquad \Rightarrow \qquad Y = \mathrm{const.}$$

Questions, Comments & Some Future Directions

- Baryonic operators & the “baryonic” R-symmetry contribution.
- Complete chiral ring/spectrum?
- Complex structure deformations (via p-forms).
- Elliptic genera?
- More general: fluxes, geometries, … (results on Betti multiplets exist.)
- $C(M_{SE(5)})$ vs. $C(M_8)$?

Thanks!